Journal of Economic Growth
https://doi.org/10.1007/s10887-024-09248-w

Has the real rate of return "depreciated"?

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Accepted: 23 September 2024

Abstract
The average depreciation rate in the United States has increased since the 1970s, a pattern most likely matched in other advanced economies. We argue that a higher depreciation rate has reduced the risk-free interest rate. We do so in a quantitative overlapping-generations model which allows for risk-premia and market power. We show that the importance of the rate of depreciation on the risk-free interest rate depends crucially on the elasticity of intertemporal substitution as well as the size of market power. Our calibrated model shows that higher depreciation plausibly reduced the risk-free rate by 30 basis points over the past half century. We contrast our results with models using a representative-agent framework (Farhi and Gourio in Accounting for macro-finance trends: market power, intangibles, and risk premia. Brookings papers on economic activity, 147, 2019) which typically do not find a role for the rate of depreciation.

Keywords The real rate of interest · Capital depreciation · Growth theory · Savings motive

JEL Classification E43 · O4 · E13 · E21

1 Introduction

During the past fifty years, the natural real rate of interest in Advanced Economies has declined by about 300 bps (Rachel & Summers, 2019).[1] Concurrently, there has been a consistent decrease in real rates for secure assets. Fundamentally, this trend might be the result of increased aggregate savings, reduced aggregate investments, or a mix of both. Our analysis introduces a previously overlooked but potentially important element: an

[1] Rachel and Summers (2019) estimate the natural real rate of interest as the interest rate that is consistent with output at its potential structural level and constant inflation.

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escalating rate of capital depreciation. We utilize an overlapping-generation framework and demonstrate that the more typical representative-agent framework a priori rules out a role of depreciation on the real rate of interest.

As documented in the following, the average rate of capital depreciation appears to have risen by slightly more than 1 percentage point in the United States over the last half century. The best cross-country data available indicates a similar trend in other Advanced Economies. Our analysis, utilizing comprehensive data from the U.S. Bureau of Economic Analysis, reveals that this increase in depreciation is widespread across various sectors. It remains unclear how much of the observed increase in depreciation is due to physical depreciation and how much is due to revaluation, although intuition may perhaps suggest the latter is more important when it comes to IT-related assets. We demonstrate that an increase in depreciation rates will lower the steady-state risk-free rate, regardless of its cause.

To support this claim, we develop an overlapping-generations model of a closed economy. Our model features risky investment, market power, and differential (exogenous) technological progress for consumption and investment goods. The model is designed to capture three key mechanisms. Firstly, it assumes a quicker pace of (exogenous) technological advancements in investment goods compared to consumption goods, leading to a steady decline in the relative investment prices. Secondly, the user-cost of capital depends on the cost of borrowing (i.e., the real rate of return), physical depreciation, and the rate of change in the relative price of investment. When the relative price of investment goods continuously declines (or the rate of physical decay goes up), the user-cost of capital increases. The reason why prices matter is because the act of investment involves a capital loss: the price at which the capital good is purchased is higher than the later resale value. Hence we capture faster depreciation as *either* greater physical decay *or* a faster rate of investment good deflation. Third, with overlapping generations, households have a life-cycle motive for savings. In our baseline model, we have a two-period life, but in the quantitative setting we allow for arbitrary length of lifespan and work life. This creates an upward-sloping supply curve of savings as a function of the risky return on assets.

These three elements interact in the following way. Assume a scenario where the rate of technological advancement accelerates for investment goods relative to consumption goods, resulting in a faster relative price deflation for investments. To fix ideas suppose that this experiment leaves the steady state growth rate of the economy, which is a combination of technological change for the consumption and investment goods, unaffected. In such a case, an increase in economic depreciation reduces the demand for capital as the user cost of capital increases. With an upward-sloping supply of savings, this lowers the risky rate and increases the gap between the marginal product of labor and the risky rate. The magnitude of the decline in the risky rate depends on the slope of the supply of savings, which we show depends crucially on the elasticity of intertemporal substitution and the extent of monopoly power in the economy. We contrast this with a representative agent model where the supply of savings is horizontal and changes to the rate of depreciation do not affect the risk-free rate.

The production side of our economy closely mirrors that of Farhi and Gourio (2019, henceforth FG). Like them, we include risky returns, monopoly power, and different rates of technological growth in an analogous manner. With risky returns, it is necessary to distinguish between the risky rate of return and the risk-free rate, and the spread between the two is determined by a combination of risk-preferences and riskiness of the asset. Since the production side of the models is identical, so is the demand for physical capital as a function of the risky rate. With identical production structures, we can focus on the crucial distinction between the two models, which is the supply of physical capital. In both models, the demand comes from firms that rent physical capital. In FG, the supply curve comes from the Euler equation of the representative agent, which creates a horizontal supply of capital, and does not permit depreciation to affect the risky rate. In our model, in contrast, savings come from agents who wish to save for future consumption. They can do so in two manners: By investing in physical capital, which is then rented by the firms, or by buying equity in the same firms. With monopoly power, firms receive monopoly rents, which they pay out as dividends. Equity is valued as the net present value of future dividend payments. The supply curve of physical capital is thereby the combination of two things: the savings of current workers and retirees and the fraction they choose to invest in physical capital. This creates a supply curve of physical capital that is generally not horizontal but depends on two factors: First, the elasticity of intertemporal substitution (EIS). If this elasticity is low, savings are only mildly affected by the risky rate. This by itself creates a steep supply curve and thereby a large effect of higher depreciation on the risky rate. However, with equity, there is an additional effect: A lower risky rate increases the net present value of dividends, and thereby the value of equity. This directs more savings towards equity and reduces the supply of capital. This flattens the supply curve of physical capital and reduces the effect of depreciation on the risky return.[2]

The gap between risk-free and risk-free rates is shaped primarily by risk factors and individual risk preferences. Thus, any changes that arise from other variables, including depreciation, simultaneously affect both rates. Consequently, any alteration of the risky rate also impacts the risk-free rate, maintaining their relationship.

We calibrate our model to the US data from 1984 to 2000, which is the time period where the rate of depreciation grew the most. We use nine equations to match nine parameters exactly. Based on these parameters, we find that the observed increase in deprecation of around 125 basis points translates into a decline in the risk-free rate of around 30 basis points for an elasticity of intertemporal substitution of 0.25.[3] These are significant effects in light of the observed decline in the natural real rate of return of about 300 bps (Rachel & Summers, 2019). There is considerably disagreement about the size of the EIS. In the following, we discuss this literature and argue for a low EIS between 0 and 0.5. In a competitive economy, the shift in savings towards equity would not have been present and the effect on return consequently around 90 basis points, illustrating the importance of monopoly power. The increase in the rate of depreciation was the highest in the period until the 2000s, and it has been relatively stable since while the risk free rate has continued to decline. Consequently, we focus our analysis on the time period leading up to 2000 and do not claim that the rate of depreciation has contributed to the decline in the rate of interest over the past 20 years.

We employ a Cobb–Douglas production function. This ensures that a familiar mechanism, linking a *change* in the relative price—as opposed to a change in the rate of decline of prices—of investment to the real rate, is not present: Holding the marginal product constant, cheaper investment goods means that a given amount of savings buys more

[2] Moll et al. (2022) develop a model with an upward-sloping supply curve of capital in the interest rate arising from a specific dissipation shock to wealth accumulation and argue that a broad class of models would feature upward- sloping supply curves. Though our focus here is on the savings motive, other models with an upward sloping supply curve would also find that a higher depreciation rate lowers the real interest rate.

[3] The effect would be higher for a lower EIS, but less than 10 basis points for an EIS of 2

investment, which increases the return to investment, but holding savings constant, more investment lowers the marginal product of capital, which reduces the return on investments. If the elasticity of substitution between capital and labor is smaller than one, the real rate is reduced in equilibrium (e.g., Sajedi and Thwaites, 2016). With identical Cobb–Douglas technologies, these effects cancel out. Canceling the familiar effects of a change in the relative price of investment allows us to focus on the effects from changes to the *rate* of decline in the price of investment goods.[4]

The paper is related to the recent literature that discusses plausible explanations for the observed decline in real rates of interest over the past half century. Several contributing forces have been put forward. Useful overviews of the literature are found in Rachel and Smith (2017), Rachel and Summers (2019), and Kiley (2020). The contribution of the present paper is to explore the relevance of capital depreciation, a factor that has so far seemed neglected. Similarly related are contributions that aim to explain the gap between the marginal product and the real rate, e.g. via rising market power or rising risk premia; Eggertsson et al. (2021) contains an overview. Rising depreciation is a mechanism that has also been left unexplored in this literature. One notable exception is FG who do explore the importance of depreciation for gap between the marginal product and the risk-free rate, concluding it has been unimportant. As we argue below, the reason for this is twofold: the representative agent framework implies a horizontal savings schedule, which our OLG model does not. Second, they compare 1984–2000 to 2001–2016 and infer relatively little increase in depreciation (physical + economic) between the two periods. We directly calculate the average depreciation rate over a longer time period from 1970 onward, where the rate of depreciation has increased by more than 100 basis points.

The paper proceeds as follows. In the next section, we document that the average rate of depreciation has increased. Section 3 demonstrates that for our purposes physical depreciation and economic depreciation from price deflation are equivalent. Section 4 provides a two-sector OLG and derives our analytical results. Section 5 extends the model to longer life expectancy and working life and performs the quantitative assessment.

2 Aggregate movements in the rate of depreciation

2.1 Measuring capital depreciation

From a national account perspective, "depreciation" is defined as the change in the value of a capital good associated with the aging of the asset (Fraumeni, 1997). When a capital asset ages, its value may change for several reasons. For one, physical wear and tear, which cause the productive capacity of a capital good to decline, make it less valuable as it ages. In addition, the value of an asset can change due to inflation, revaluation, or other factors that may be correlated with the age of the asset.

In practice, the rate of capital depreciation can be estimated for an individual type of capital good using the regression-based approach pioneered by Hulten and Wykoff (1981). By employing data on the resale price of assets, the effect of aging can be separated from

[4] While the mechanism discussed in Sajedi and Thwaites (2016) and the one in focus here are not mutually exclusive in their contribution to a decline in the real rate of interest only the depreciation channel produces a gap between the real rate and the marginal product.

pure time effects caused by inflation, and if data are available, the "vintage" effect can also be controlled for.[5]

The Bureau of Economic Analysis (BEA), for example, distinguishes between more than 250 different asset types. While the depreciation pattern for a particular vintage of a capital good is assumed to be constant over time, the depreciation profile may differ across different vintages of capital goods (BEA, 2003, p. 29).

Ultimately, BEA and other statistical agencies use the estimates for depreciation to construct net capital stocks and thus consumption of fixed capital in national accounts (Katz & Herman, 1997). Combining depreciation rates with investment data, the perpetual inventory method is employed to construct net capital stocks. The net stock of capital in a particular year is the difference between the accumulated past gross investment and the value of the accumulated depreciation, and the calculation is conducted at the type-of-asset level of detail.

Finally, to calibrate the average depreciation rate many macro applications have traditionally used the weighted average of the underlying (estimated) depreciation rates where the individual weights are the *real shares* of the individual capital stocks. This can be accomplished by "inverting" the aggregate capital accumulation equation where investments and capital are at constant prices (Cooley & Prescott, 1995):

$$\delta_t = \frac{I_t - \Delta K_t}{K_{t-1}}.$$

However, the procedure has a couple of drawbacks. First, when using real shares as weights, the time path of the average depreciation rate becomes sensitive to the choice of the base year, just as the average growth of fixed-price GDP is sensitive to the choice of the base year. Specifically, if high depreciation assets have exhibited declining prices over time (e.g., computers), and the final year is used as a base year, the real share in the initial year is lowered, which mechanically tends to produce a positive trend (Oulton & Srinivasan, 2003). Second, if investments are chain-weighted, it is no longer true that the above approach produces a weighted average of the underlying depreciation rates (Whelan, 2002). Once again the "traditional" approach might lead to an artificially upward trending average depreciation rate (Whelan, 2002). To avoid these drawbacks Whelan (2002) and Oulton and Srinivasan (2003) recommend using *nominal shares* as weights when one calculates the average depreciation rate. We follow this approach when looking at the BEA data for the US below.

There is a varied literature on the measurement of the depreciation rate, in particular using US and Canadian data (BEA, 2003; Patry 2007, respectively). Tevlin and Whelan (2003) establish that the depreciation rate has increased substantially due to the increased reliance on computers, and more recent work estimates the rate of depreciation of R &D (de Rassenfosse & Jaffe, 2017).

[5] The precise shape of the link between the age of an asset and its price can in addition be used to assess which type of depreciation seems to be occurring. For example, if capital depreciation is geometric one would expect the resale value of the asset to decline geometrically with the age of the asset; geometric depreciation is often difficult to reject (Hulten and Wykoff, 1981).

2.2 The evolution of average depreciation: a cross-country perspective

We start by exploring the evolution of average depreciation for the bloc of Advanced Economies (AEs). Data on average depreciation rates come from Penn World Tables (PWT, Feenstra et al., 2015).[6] In the cross-country context, we focus on the GDP-weighted average depreciation rate for the group of Advanced Economies (AE), as defined by the IMF, from 1970 until today. In addition to the weighted average, we also calculate the simple average and the median.[7] We focus on AEs because Rachel and Summers (2019) recently estimated the natural rate of interest for this group, as noted in the Introduction, documenting a decline since 1970.

The result is shown in Fig. 1a. While the average depreciation rate is relatively flat from 1970 to 1990, it has increased by around 100 basis points since 1990. As can be seen, the simple average and the median move in a similar way, suggesting that this pattern is pervasive for the group of countries in focus. With an eye to the analysis in the next section, where we focus on the US, panel b of the figure depicts the evolution of average depreciation in the US according to PWT. The path is similar to that detected for the AE group as a whole, albeit the recent increase is greater than that for the AE group.

Why has the depreciation rate increased? In the case of the PWT, where the capital stock comprises nine types of capital with depreciation rates assumed constant and identical across countries, the reason is a given: the composition of the capital stock has changed. Over time an increasing fraction of the capital stock consists of short-lived assets, such as ICT and software. At the same time, it is important to note that PWT calculates the average depreciation rate using real shares as weights (e.g. Inklaar et al., 2019). As outlined above (and discussed in Oulton and Srinivasan, 2003), this tends to mechanically produce upward trends in the deprecation rate. This is perhaps most easily illustrated by ICT capital in the 1990s. Measured in today's dollars the real stock of ICT capital in the 1990s was very small due to the large subsequent declines in ICT prices. The average depreciation rate weighted by real capital will therefore put almost no weight in the 1990s on the relatively high depreciation rate of ICT. However, from the perspective of agents making investment decisions in the 1990s what mattered was prices at the time, that is, nominal values. In Appendix A we elaborate on this: we consider a counter-factual series where both the nominal capital shares and the depreciation rate for each asset type are constant. From the

[6] Ideally one would like to use detailed national accounts data for each country. But since the practices regarding depreciation are not fully aligned across statistical agencies (some may use linear depreciation for parts of the period in focus, for example), we resort to Penn World Tables (PWT, Feenstra et al., 2015) where the average depreciation rate is computed at a consistent basis. The cost is a smaller selection of individual assets; PWT distinguishes between nine different asset types. Depreciation is assumed to be geometric and constant across countries and time (cf Inklaar et al, 2019, Table 3).

[7] The PWT does not have information on GDP and depreciation rates for all AEs throughout all years since 1970. Therefore, we exclude a few smaller economies. These countries constitute less than 4 percent of GDP in 2017.

perspective of agents in the economy the depreciation rate would be constant, but using the real stock of assets as weights creates a growing depreciation rate.[8,9]

To better understand the movements in average depreciation, we, therefore, turn to data for the US. This change in perspective will allow us to explore the evolution of average depreciation using nominal shares as weights (Whelan, 2002; Oulton & Srinivasan, 2003). It will also allow us to perform a detailed decomposition analysis designed to shed light on the underlying drivers of the long-run evolution of average depreciation.

2.3 The evolution of average depreciation: US data

Based on BEA data, Fig. 2 shows the evolution of the average depreciation rate for the United States from 1970 to 2020 (weighted by nominal capital). As is clear, the overall increase in average depreciation weighted by nominal stocks is a bit more modest than suggested by PWT data weighted by real stocks. The time path is also somewhat different. Using the BEA data, depreciation rises from the early 1970s until the early 1990s and stays more or less flat until 2020, compared with a more gradual increase for the PWT data. Quantitatively the average depreciation rate rises by about 125 basis points from 1970 to 2000 with a relatively flat path from 2000 to 2020.

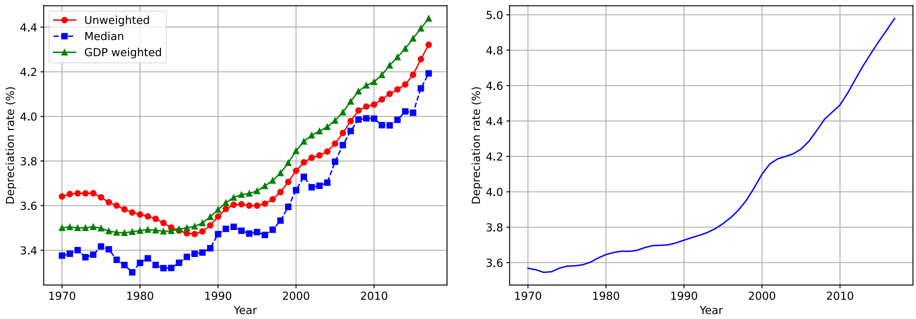
Using the richer BEA dataset, it is possible to analyze the proximate sources of this increase at a higher level of resolution. Specifically, to explore the source of rising depreciation we use the detailed subdivision of 72 asset types. We next calculate the overall depreciation rate as $\delta_t = \sum_i \delta_{i,t} K_{i,t} / K_t$, where $K_{i,t}$ is the nominal (current-cost) stock of private fixed asset of type i (K_t is the total stock) and $\delta_{i,t}$ is the depreciation rate (deprecation divided by stock of capital for each asset type). Finally, we perform a decomposition of the change in the deprecation rate between period $t - s$ and t as:

$$\delta_t - \delta_{t-s} = \underbrace{\sum_i \delta_{i,t-s} \left(\frac{K_{i,t}}{K_t} - \frac{K_{i,t-s}}{K_{t-s}} \right)}_{\textit{Reallocation}} + \underbrace{\sum_i \frac{K_{i,t}}{K_t} \left(\delta_{i,t} - \delta_{i,t-s} \right)}_{\textit{Change within asset type}},$$

where the first term gives changes in the aggregate depreciation rate from differential growth in asset classes with different depreciation rates, and the second term considers changes in the depreciation rate within asset types.

[8] More formally: Consider an economy with two types of assets and a Cobb–Douglas aggregator of the two. The capital types are ICT where prices decline with $\Delta > 0$ each period such that the price at time t is $P_t = (1 + \Delta)^{-t}$. Structures do not see declining prices. Assets have the same physical rate of deprecation. Suppose a steady state exists where investment in the two types grows in proportion to the economy. In such an economy, the ratio of each nominal stock of capital to output will be constant. However, the ratio of the real stock of ICT capital to production will increase at a rate Δ. This implies that when using the real stock of capital when weighing the aggregate depreciation rate (physical + economic), one puts increasing weight on ICT and consequently measures an increase in the depreciation rate. Furthermore, the aggregate depreciation rate for any given year will depend on the choice of base year for the price index. The nominal stock of capital does not suffer from this built-in bias. Furthermore, for agents' investment choices, it is the nominal stock of capital that matters.

[9] Though the PWT relies on 9 underlying asset categories, the publicly available data only has four: Structures, machinery, transport equipment, and others. Since the detailed BEA data demonstrate that software and IT are particularly important, we do not perform an exercise in which we weigh the PWT data by nominal assets.



(a) Advanced Economies

(b) United States

Fig. 1 The depreciation rate for advanced economics (weighted by real capital. *Source*: PWT 9.1)

Figure 3a performs this decomposition for each one of the decades from 1970 to 2020 as well as the whole period. Though there has been substantial variation over the decades, the two terms are of roughly equal importance.[10] Since the 72 asset types used for the calculations here are themselves aggregates of finer asset types, this estimate presents a lower bound on what the reallocation effect would be from a finer disaggregation. ICT and software are important drivers of the increase in depreciation: Whereas the increase in depreciation across all assets is 1 percentage point, it is 0.8 when excluding ICT and 0.2 when excluding both ICT and software.

As can be seen from the figure, the analysis suggests a roughly 50/50 split between reallocation and change within asset types. The two categories have seen the largest increase in the rate of depreciation are computers and pre-prackaged software with, respectively, 23.7 and 21.2 point increase in the rate of depreciation from 1970 to 2020. These calculations complement existing studies that have focused on the impact on aggregate depreciation from the rise of particular asset classes such as ICT (Tevlin & Whelan, 2003) or R &D and intangibles (Corrado et al., 2005)

We perform an analogous analysis based on industry composition. Panel b of Fig. 3 shows a decomposition using the same method across 18 industries. The panel shows that the entire change in depreciation comes from within-industry changes with a minor negative contribution from reallocation. As a result, the underlying changes in the capital stock which account for the rise in depreciation is a pervasive phenomenon across sectors. To the best of our knowledge, this is a novel finding. Though not the main focus of the present paper, understanding this pattern deeper would be an interesting focus for future work.

2.4 Discussion

The average rate of capital depreciation using the nominal stock of capital has risen since 1970 in the US. The data from the PWT show a similar trend for the block of AEs. Though this data weighs the depreciation rate by the real stock of capital and consequently, has a

[10] The negative reallocation effect from 2000 to 2009 is primarily due to a peak in ICT/software capital around the Dot-com boom around the year 2000. There is some contribution from growing real estate capital until 2009 as well.

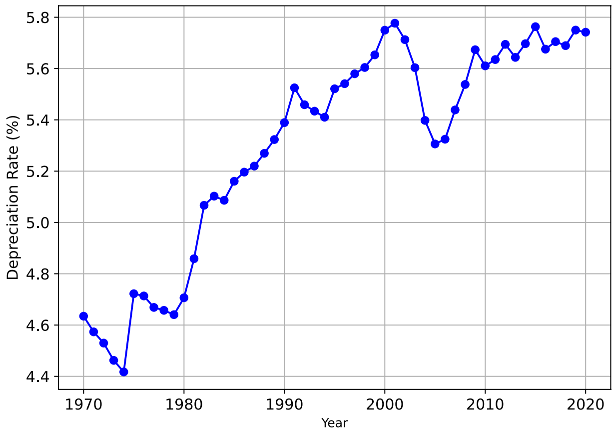


Fig. 2 The depreciation rate for the United States (weighted by nominal capital. *Source*: Bureau of Economic Analysis)

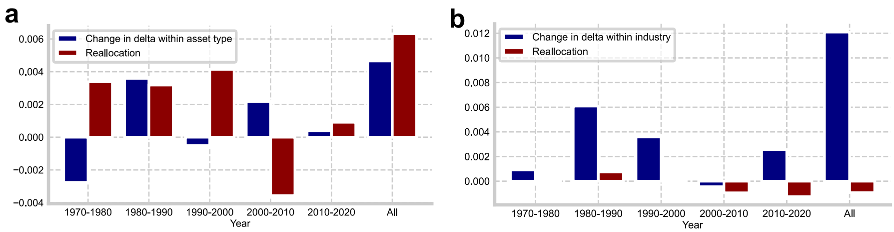


Fig. 3 Decomposition of change in depreciation rate—by decades and whole period. *Note*: The accumulation of changes over decades does not add up to change over the whole period

built-in bias towards an increase in the rate of depreciation, the similar patterns in cross-country data suggest a pervasive phenomenon. Baldwin et al. (2015) also find an increase in the aggregate depreciation rate in Canada, primarily due to compositional effects.

As discussed above, average depreciation rates are affected by both physical "wear and tear" and revaluation; if the (relative) price of used capital declines, it shows up as faster capital depreciation. When the resale value of capital falls it may, however, also reflect that investment goods are firm- or sector-specific. Studying aerospace plant closings, Ramey and Shapiro (2001) find evidence that investment specificity appears to be important in practice. But since inter-sectoral reallocation seems to be of minor importance to the rise in average depreciation this latter channel is probably not paramount to the issue in hand. This leaves accelerated physical decay and revaluation.[11]

There is no general way in which to fully separate "wear and tear" from revaluation (Hall, 1968), so the data may be taken to imply that over time the capital stock increasingly consists of capital types featuring faster physical depreciation, revaluation or both. In

[11] The study by Ramey and Shapiro (2001) also contributes to the literature on capital depreciation in that they have access to the actual purchase price of the equipment, which otherwise is assumed in the literature to be identical to the list price. Moreover, by studying a natural experiment (plant closings) their sales data should be free of the "lemons problem" which refers to the concern that part of the price reduction that the literature classifies as depreciation may be due to selection. Reassuringly their estimates are fairly similar to the existing estimates in the literature.

the context of IT equipment, the revaluation channel is most likely relatively more important (Geske et al., 2007), although age-related effects also seem to matter (Doms & Lewis, 2005).[12]

Before proceeding to the full model, we demonstrate the equivalence of physical and economic depreciation (through revaluations).

3 Physical depreciation and revaluation

Before we proceed with the theoretical model, we quickly discuss the distinction between physical depreciation and revaluation (Fraumeni, 1997). Though typically depreciation is modeled as physical, as discussed above, the observed rise in depreciation is equally likely to be the result of revaluation considering the key role played by IT, software, etc. in the observed increase in average depreciation. Karabarbounis and Neiman (2014) show an accelerating decline in the global price of investment goods from 1980 since when real prices have declined by around 2 percent a year to 2010. This would depreciate the value of existing capital goods.

To see the influence from revaluation in a simple way, suppose that we distinguish between investment and consumption goods with different price trajectories. We normalize the price of consumption goods to 1, and, to simplify matters, consider a one-period problem facing a profit-maximizing representative firm. The firm can rent capital in a competitive market at the rate R_t.

Profits are therefore given by:

$$\Pi = F(K_t, A_t L_t) - w_t L_t - R_t K_t,$$

with first order condition:

$$F_K = R_t K_t.$$

Now, consider the a young person who wishes to save one unit of the final good for the future in period $t-1$. This person can do so by buying capital stock at the prevailing price p_{t-1} and renting it the following period to the firm. One unit of the final good gives $1/p_{t-1}$ units of capital which gives a return of:

$$1 + r_t = R_t/p_{t-1} + (1-\delta)p_t/p_{t-1}.$$

This expression has two terms: First is the rental rate paid by the firm. Second it the value of the capital in the following period. This consists of the remaining $(1-\delta)/p_{t-1}$ physical units of capital now priced at (the lower) p_t.

Combining the two we find a relationship between marginal product of capital and the return on savings as:

[12] As pointed out by the authors, this channel undoubtedly captures more than physical decay. E.g., if new software becomes progressively harder to run on existing IT equipment, this will show up as an "age effect", which illustrates the point that economic and physical depreciation is hard to disentangle in practice.

$$F_K = p_{t-1}\left[1 + r_t - \frac{(1-\delta)}{p_{t-1}/p_t}\right],$$

where $p_t/p_{t-1} < 1$ with declining prices. With constant capital prices at $p_t = 1$, this condition reduces to the familiar: $F_K = MPK = r_t + \delta$. But in the two-sector setting, the (relative) price of capital reflects buying and selling of capital may involve either capital gains or losses. In the case where the price of capital is declining over time, relative to the price of consumption, the transaction involves a capital loss which raises the user-cost of capital just like higher physical capital depreciation does. From a partial perspective, therefore, there is no difference between an increase in δ or a fall in p_t/p_{t-1}; both will serve to reduce the demand for capital for a given r_t.[13] As discussed in the last section, the rise in NIPA average depreciation is most likely caused by either rising physical depreciation (δ), revaluation ($p_t/p_{t-1} < 1$), or both. Though the following model will have both monopoly power and risky returns, this partial equivalence between physical deprecation and economic depreciation (revaluation) will still be present.[14]

From a broader perspective, however, there is a difference in that the fall in the price of capital is endogenous. It might, for instance, be caused by a faster rate of technological changes in the capital goods-producing sector than what prevails in the consumption goods sector. A faster decline in the (relative) price of capital, therefore, reflects a faster rate of technological change which will matter to capital accumulation in its own right via the long-run growth rate of the economy. But a similar outcome to the one described in the one sector setting can be obtained if total growth of the economy is held constant by changing the sources of technological change.

Below we construct a two-sector model where both the consumption good and the investment good are produced with the final good, and there is technological progress in both the production of the final good and the investment good.[15] The steady-state growth rate of the economy combines these two. An increase in the growth rate of technology for investment goods will increase the overall growth rate of the economy, which has not generally been observed. However, a shift in technological growth from consumption goods to investment goods leaves the growth rate of the overall economy constant but drives the prices of investment goods down thus producing the effect on the user-cost of capital discussed above.[16] In this case, the remarks above will carry over: faster economic depreciation will lower the real rate of return in the OLG setting, but not in the model with a representative agent.

[13] Note, what matters is here is the deflation in prices of capital, not the level per se. Sajedi and Thwaites (2016) use a framework with a CES production function and demonstrate how the change in the *level* of the relative price of capital influences the return to capital.

[14] Fraumeni (1997) distinguishes between revaluation due to a obsolescence or changes or changes in asset prices. Our framework does not distinguish between the two.

[15] Alternatively, we could have built a vintage capital model along the lines of Phelps (1962). This will give rise to the same result.

[16] This is related, but distinct, from a recent literature arguing that technological change has shifted in favor of automation without increasing the overall growth rate of the economy (Acemoglu and Restrepo, 2019; Hémous and Olsen, 2022). *Automation*, however, is the replacement of labor tasks with tasks performed by capital, whereas faster technological growth in investment goods is the more rapid replacement of capital by new vintages of capital.

4 The model with two generations

We consider a closed overlapping-generations economy. Time is discrete $t = 1, 2...\infty$. The economy consists of two sectors that produce consumption goods and investment goods, respectively. The price of the consumption good is normalized to one. Both sectors experience exogenous technological change, but the growth rate is stochastic as specified below. The population grows at the rate g_L. The production side of the economy mirrors that of Farhi and Gourio (2019, FG), but whereas they use a representative agent framework our model features overlapping generations.

4.1 Consumers

For simplicity we first consider a model where agents live for two periods. When we bring the model to the data and relate it to the quantitative work of FG we allow for lifespans of arbitrary length. In the first period of life, workers supply one unit of labor inelastically for which they receive a wage, w_t. During old-age, individuals consume the returns on their savings. To flexibly allow for the role of risk, we follow FG and assume Epstein-Zin preferences such that the utility of a young person (in period 1 of their life) is given by:

$$V_{1,t} = max_{c_{1,t}} \left((1-\beta) c_{1,t}^{1-\sigma} + \beta E_t (V_{2,t+1}^{1-\theta})^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}}. \tag{1}$$

Without uncertainty this reduces to the standard specification of discounted utility.

Here $c_{1,t}$ and $c_{2,t}$ are the consumption of, respectively a young and an old person in period t. σ is the inverse of elasticity of intertemporal substitution (EIS), and θ is the coefficient of relative risk aversion. When $\sigma = \theta$ the model features standard discounted expected utility expression.

The utility of an old person, where uncertainty has been resolved, is:

$$V_{2,t} = (1-\beta)^{\frac{1}{1-\sigma}} c_{2,t}. \tag{2}$$

In anticipation of the equilibrium structure of the model, we solve the savings problem of a young person who only has access to a savings technology with stochastic return of $(1 + r_{t+1}^*)e^{\chi_{t+1}}$,where χ_{t+1} is a stochastic shock to returns and is i.i.d.. We let $E_t e^{\chi_{t+1}} = 1$ such that the expected gross return on this asset is $1 + r_{t+1}^*$. Let s_t be the savings of a young person. The budget constraints are then:

$$c_{1,t} + s_t = w_t, \tag{3}$$

$$c_{2,t+1} = e^{\chi_{t+1}} (1 + r_{t+1}^*) s_t, \tag{4}$$

such that a young worker chooses between savings and consumption and the old person only consumes their savings.

Combining the budget constraints (3 and 4) with second period utility (2) and rewriting the the utility function of the young person (1) we get the young person's maximization problem:

$$V_{1,t} = max_{c_{1,t}} \left((1-\beta) c_{1,t}^{1-\sigma} + \beta (1-\beta)(w_t - c_{1,t})^{1-\sigma} \left(1 + r_{t+1}^*\right)^{1-\sigma} E_t (e^{(1-\theta)\chi_{t+1}})^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}}$$

which has a solution of

$$s_t = \frac{1}{1 + \beta^{-1/\sigma}(1 + r^*_{t+1})^{\frac{\sigma-1}{\sigma}} E_t(e^{(1-\theta)\chi_{t+1}})^{\frac{\sigma-1}{\sigma}\frac{1}{1-\theta}}} w_t = \hat{s}(r^*_{t+1})w_t, \tag{5}$$

where $\hat{s}(r^*_{t+1})$ is the savings rate out of present income. With χ_{t+1} i.i.d. the function $\hat{s}$ is time-independent.

Further, consumption in the first period of life is given by

$$c_{1,t} = (1 - \hat{s}(r^*_{t+1}))w_t, \tag{6}$$

and the (stochastic) consumption in second period is:

$$c_{2,t+1} = e^{\chi_{t+1}}(1 + r^*_{t+1})\hat{s}(r^*_{t+1})w_t \tag{7}$$

With $\sigma \to 1$, preferences are logarithmic and the savings rate out of income is $\hat{s} = \beta/(1+\beta)$ and independent of expected return and the variance of χ_{t+1}. Consider an increase in the expected return, r^*_{t+1}, which lowers the relative price of future consumption. When the EIS is less than 1 ($\sigma > 1$), this substitutes consumption towards the young age, and lowers savings: $\hat{s}' < 0$. Further, a riskier χ_{t+1} for constant $E_t e^{\chi_{t+1}}$ reduces $E_t(e^{(1-\theta)\chi_{t+1}})^{\frac{1}{1-\theta}}$, and for $\sigma > 1$ this increases savings. These effects are reversed when $\sigma < 1$.

Given the uncertainty, assets with different risk profiles will have different expected returns. In particular, the risk-free rate will be lower than r^*_{t+1}. To price these assets we employ a convenient tool from finance, the stochastic discount factor (Epstein & Zin, 2013). It is a generalization of the relative marginal utility of consumption between periods:

$$M_{t+1} = \beta\left(\frac{c_{2,t+1}}{c_{1,t}}\right)^{-\sigma}\left(\frac{V_{2,t+1}}{E_t(V_{2,t+1}^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta} = \beta\left(\frac{c_{2,t+1}}{c_{1,t}}\right)^{-\sigma}\left(\frac{c_{2,t}}{E_t(c_{2,t}^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta} = \beta\left(\frac{c_{2,t+1}}{c_{1,t}}\right)^{-\sigma}\left(\frac{e^{\chi_{t+1}}}{E_t(e^{(1-\theta)\chi_{t+1}})^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}. \tag{8}$$

The first equality is the general expression for the stochastic discount factor with Epstein-Zin preferences (see FG). The second equality utilizes the structure of the two period model: $V_{2,t} = (1-\beta)^{\frac{1}{1-\sigma}} c_{2,t}$. The third equality substitutes for $c_{2,t+1}$ using Eq. (7).

To see the utility of this, consider an asset with a stochastic outcome Ω_{t+1}. The stochastic discount factor can price any such asset, traded or not, as follows:

$$P^{\Omega}_t = E_t\left[M_{t+1}\Omega_{t+1}\right].$$

The intuition is that M_{t+1} records the marginal discounted utility value of a payout in each state and P^{Ω}_t is the expected value of these payouts. If the economy had no uncertainty the stochastic discount factor would be: $M_{t+1} = \beta\left(c_{2,t+1}/c_{1,t}\right)^{-\sigma}$ and the rate of return can be found from $1 = \beta(1 + r_{t+1})(c_{2,t+1}/c_{1,t})^{-\sigma}$, the familiar Euler equation. The more general expression allows for the marginal value of payouts to vary with the draw of χ_{t+1}. Naturally, the price of the asset that pays $(1 + r^*_{t+1})e^{\chi_{t+1}}$ in period $t+1$ is 1.

The risk free rate can be thought of as an asset that costs 1 in period t and pays out $1 + r^f_{t+1}$ in period $t+1$. Consequently, we can find the value of $1 + r^f_{t+1}$ which has a price of 1:

$$1 = E_t\left[M_{t+1}(1 + r^f_{t+1})\right] = E_t\left[\beta\left(\frac{c_{2,t+1}}{c_{1,t}}\right)^{-\sigma}\left(\frac{e^{\chi_{t+1}}}{E_t(e^{(1-\theta)\chi_{t+1}})^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}(1 + r^f_{t+1})\right] = \frac{1 + r^f_{t+1}}{1 + r^*_{t+1}}\frac{E_t e^{-\theta\chi_{t+1}}}{E_t(e^{(1-\theta)\chi_{t+1}})} \Leftrightarrow$$
$$1 + r^f_{t+1} = (1 + r^*_{t+1})\frac{E_t(e^{(1-\theta)\chi_{t+1}})}{E_t e^{-\theta\chi_{t+1}}}, \tag{9}$$

where we have used Eqs. (6)–(8). Consequently, the spread between the expected return on the risky asset $(1 + r^*_{t+1})$ and the risk-free rate is captured by the properties of the shock χ_{t+1} and the risk preferences θ. The fraction is always weakly smaller than one such that the risk free rate is lower than $1 + r^*_{t+1}$. Any parameter that changes $1 + r^*_{t+1}$ (other than changes to the distribution of χ_{t+1} or θ) will proportionally affect the risk-free rate. Consequently, when studying the equilibrium below, we can focus on the return on the risky asset and find the risk-free rate as a consequence of Eq. (9)

The function $\hat{s}(r^*_{t+1})$ captures the supply of capital in the economy. In the following, we specify the production side which will capture demand for capital as a function of r^*_{t+1}. For comparison with FG we closely follow their specification of the production side of the economy.

4.2 Production

The final output is produced competitively with a unit measure of types, $i \in [0, 1]$ of intermediate input.

$$Y_t = \left(\int_0^1 y_{i,t}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$ is the elasticity of substitution between intermediate inputs. The price of the final good is normalized to 1 and is used for consumption or investment such that each point in time:

$$C_{1,t} + C_{2,t} + X_t = Y_t,$$

where $C_{i,t}$ is consumption in aggregate of generation i and X_t is total final good spent on investment.

Each intermediate input is produced by a unique monopolist with the production function:

$$y_{i,t} = Z_t k_{i,t}^{\alpha} (S_t l_{i,t})^{1-\alpha},$$

where $k_{i,t}$ and $l_{i,t}$ are capital and labor input of firm i at time t. Z_t is an exogenous deterministic productivity trend that grows at g_Z. S_t is a stochastic i.i.d. shock to productivity which follows:

$$S_{t+1} = S_t e^{\chi_{t+1}}.$$

Capital is accumulated using the final good and a stochastic production function. In particular, one unit of the final good spent on investment in period t produces $Q_t e^{\chi_{t+1}}$ units of capital in period $t + 1$. The technology parameter Q_t grows exogenously at g_Q. Investments goods are produced competitively with the final good, and the price of an expected unit of capital is $1/Q_t$. Consequently, the accumulation function for the individual firm is:

$$k_{i,t+1} = \left[(1 - \delta) k_{i,t} + Q_t x_{i,t} \right] e^{\chi_{t+1}}.$$

Due to uniform production technology, the economy easily aggregates to an aggregate production function:

$$Y_t = Z_t K_t^{\alpha} (S_t L_t)^{1-\alpha},$$

where K_t is aggregate capital and L_t is the size of the young generation that grows with g_L.[17] Given demand from the final good producer, each intermediate input producer charges a markup of $\mu \equiv \epsilon/(\epsilon - 1) > 1$. With zero profits in the final good sector, the total revenue for intermediate input producers is Y_t. A fraction $\frac{\mu-1}{\mu}$ of this is the profit of the intermediate input producers. The remaining $1/\mu$ goes to labor and capital. Firms do not own the capital and hire workers at wages w_t and rent capital at the gross rental rate of R_t. All intermediate input producers have identical Cobb–Douglas production functions such that:

$$(1-\alpha)\frac{Y_t}{\mu} = L_t w_t,$$
$$\alpha \frac{Y_t}{\mu} = R_t K_t.$$

Furthermore, the law of motion for aggregate capital is:

$$K_{t+1} = \left[(1-\delta)K_t + Q_t X_t\right] e^{\chi_{t+1}}. \tag{10}$$

Consider the return on investment in physical assets. If an investor spends one unit of the final good on capital in period t, she will acquire $Q_t e^{\chi_{t+1}}$ units of capital in period $t+1$. The capital will earn $\frac{\alpha Y_{t+1}}{\mu K_{t+1}}$ per unit of production and the physical depreciation will leave a fraction $(1-\delta)$ of the capital left. The price of capital will have decreased to $1/Q_{t+1}$ units of final good the following period. Consequently, the gross return on investing in a unit of capital is given by:

$$R_{t+1}^K = \left(\frac{\alpha Y_{t+1}}{\mu K_{t+1}} Q_t + \frac{1-\delta}{Q_{t+1}/Q_t}\right) e^{\chi_{t+1}}, \tag{11}$$

where the first term is the gross return to capital and the second term corrects for both physical and economic depreciation as discussed in Sect. 3. This expression has the same risk profile, $e^{\chi_{t+1}}$, as the asset considered in Sect. 4.1. The savings function $\hat{s}(R_{t+1}^K - 1)$ can therefore be used to determine the savings behavior of young people investing in physical capital.

The physical capital is not the only asset in this economy. The intermediate input producers command monopoly profits and consequently, equity in these firms has a value. Firms pay out dividends of $D_t = (\mu - 1)/\mu Y_t$ each period. The price of this asset can also be priced using the stochastic discount factor as:

$$P_t = E_t\left[M_{t+1}(P_{t+1} + D_{t+1})\right], \tag{12}$$

which reflects that the price of equity in period t (after dividends have been paid) must equal discounted value of next period's dividends plus the price of equity next period, P_{t+1}.

[17] FG allow for a labor force participation of less than 1 but find that this has no significant effect on the real interest rate. In the present setting with overlapping generations, we could introduce a share of the young generation that does not work. This would increase notational complexity with no additional insight and we do not pursue this.

With a unit mass of firms P_t is both the price of a single firm and total value of equity. That is, firms live forever, and each period they are sold from the old to the young.

Without additional structure, it is difficult to price equity in general. In the next section, we follow FG and specify a *risky balanced growth path*, which can be solved analytically.

4.3 The risky balanced growth path

To solve the model we conjecture that at each point in time the equilibrium of the model features:

$$Y_t = T_t S_t y^*, \tag{13}$$

$$K_t = T_t S_t Q_{t-1} k^*, \tag{14}$$

$$C_{1,t} = \left(T_t S_t c_1^*\right), \tag{15}$$

$$C_{2,t} = \left(T_t S_t c_2^*\right), \tag{16}$$

with $X_t = T_t S_t x^*$ and $w_t = T_t S_t w^*$ and other aggregate variables analogously defined (the definition of k^* using Q_{t-1} is notationally convenient and is immaterial). The price of equity will likewise follow $P_t = T_t S_t p^*$. This implies that output of the economy, Y_t, will follow a deterministic path T_t and a stochastic path governed by the realizations of χ_t and thereby S_t. The deterministic growth rate, T, is given by $T_t = L_t Z_t^{\frac{1}{1-\alpha}} Q_t^{\frac{\alpha}{1-\alpha}}$ and grows as:

$$1 + g_T = (1 + g_L)(1 + g_Z)^{\frac{1}{1-\alpha}}(1 + g_Q)^{\frac{\alpha}{1-\alpha}}.$$

Using the structure of the risky balanced growth path along with the price of equity (Eq. 12) we get:

$$P_t = E_t\left[M_{t+1}(P_{t+1} + D_{t+1})\right] \Leftrightarrow 1 = E_t\left[M_t\left(1 + \frac{\mu - 1}{\mu}\frac{y^*}{p^*}\right)(1 + g_T)e^{\chi_{t+1}}\right],$$

from which it follows that the riskiness of equity investments equal those of capital from Eq. (11) (they are scaled by $e^{\chi_{t+1}}$). Consequently, the return must be the same. Using the expression for the stochastic discount factor, M_{t+1}, from Eq. 8 we can further find:

$$p^* = \frac{(1 + g_T)}{r^* - g_T}\frac{\mu - 1}{\mu}y^*, \tag{17}$$

where r^* is the expected return on both equity and physical capital. This is a Gordon formulae: The value of equity equals the discounted value of future profits where the discount factor is the expected return on the risky asset corrected for the growth of output. While the dividend share of output remains constant at $(\mu - 1)/\mu$, the value of equity relative to the output is not fixed, but depends on the r^*. This will be important in what follows.

Finally, we use the structure of the risky balanced growth path along with (11) to find the value of y^*/k^*:

$$1 + r^* = \left(\frac{\alpha y^*}{\mu k^*} + \frac{1 - \delta}{(1 + g_Q)} \right) \Leftrightarrow r^* = \left(\frac{\alpha y^*}{\mu k^*} - \frac{\delta + g_Q}{1 + g_Q} \right)$$

Section 4.1 solved for the savings of the young as a function of the return on the risky asset r^*. This section derived expressions for the two possible assets to invest in, physical capital and equity. In the following section we solve for the equilibrium and thereby determine the risky rate r^*. We derive the derivative of r^* as a function of $(\delta + g_Q)/(1 + g_Q)$, the total rate of depreciation.

4.3.1 Equilibrium in the savings market

Each period the young use their savings to purchase the two assets from the old. The remaining savings are used for new investment. Consequently, the equilibrium must be:

$$\hat{s}(r^*) \frac{1 - \alpha}{\mu} Y_t = P_t + \frac{(1 - \delta) K_t}{Q_t} + X_t = P_t + K_{t+1} Q_t^{-1} e^{-\chi_{t+1}}, \tag{18}$$

where the left hand side is total savings by the young measured in units of the final good. The right hand side uses Eq. (10). The multiplication by $e^{-\chi_{t+1}}$ comes from the fact that the value of capital the following period is stochastic and Q_t^{-1} is the price of capital and investment in period t.

With this in hand we proceed with the following Proposition which establishes the equilibrium y^*/k^* and the derivative wrt $\delta^{TOT} \equiv (\delta + g_Q)/(1 + g_Q)$.

Proposition 1 *The equilibrium level of output to capital y^*/k^* and the expected return on capital r^* are given as the solution to*:

$$\frac{y^*}{k^*} = \frac{(1 + g_T)}{\left\{ \hat{s}(r^*) \frac{1-\alpha}{\mu} - \frac{(1+g_T)}{r^* - g_T} \frac{\mu - 1}{\mu} \right\}} \tag{19}$$

$$1 + r^* = \left(\frac{\alpha y^*}{\mu k^*} + \frac{1 - \delta}{1 + g_Q} \right), \tag{20}$$

where the derivative of the expected return on capital with respect to total depreciation, δ^{TOT}, (holding total growth of the economy, g_T, constant), is given by:

$$\frac{dr^*}{d\delta^{TOT}} = - \frac{1}{\left\{ 1 + \frac{(\alpha/\mu)}{1+g_T} (y^*/k^*)^2 \left(\hat{s}'(r^*) \frac{1-\alpha}{\mu} + \frac{(1+g_T)}{(r^* - g_T)^2} \frac{\mu - 1}{\mu} \right) \right\}}.$$

(*a*) *if preferences are logarithmic, $\sigma = 1$, and market competitive, $\mu = 1$, the expected return on capital changes one-for-one with depreciation*:

$$\frac{dr^*}{d\delta^{TOT}} = -1,$$

(*b*) *if $\mu = 1$ and $\sigma > 1$, then $\hat{s}'(r^*) < 0$ and a decrease in the expected return increases savings and leads to a further decrease in expected returns*:

$$\frac{dr^*}{d\delta^{TOT}} < -1,$$

with $0 > \frac{dr^*}{d\delta^{TOT}} > -1$ *if* $\sigma < 1$

(c) *For* $\mu > 1$, *a decline in the expected return on assets diverts savings towards equity, and reduces the capital stock. In particular, for* $\sigma = 1$ *and* $\mu > 1$

$$\frac{dr^*}{d\delta^{TOT}} > -1,$$

such that there is a lower than one-to-one effect on the risky rate of return.

Proof Equation (19) follows from Eq. (18) where we impose the structure of the risky balanced growth and use (17).

The proposition demonstrates that the effect of depreciation is equivalent, regardless of whether it is physical depreciation, δ, or economic depreciation $(1 + g_Q) = Q_{t+1}/Q_t$.[18] Consider first the special case of logarithmic preferences, σ, where markets are competitive, $\mu = 1$, and there are no profits. In this case, $\hat{s} = \beta/(1 + \beta)$ and the return on capital is given by:

$$r^* = \frac{(1 + g_T)}{\frac{\beta}{1+\beta}(1 - \alpha)} - \delta^{TOT},$$

which shows that the expected return on capital changes one for one with depreciation, δ^{TOT} since y^*/k^*, is constant (Eq. 19 with $\mu = 1$ and $\hat{s} = \beta/(1 + \beta)$). The risk free rate falls in proportion as given by Eq. (9).

When $\sigma > 1$, the lower interest rate increases savings, $\hat{s}' < 0$, which reduces y^*/k^* and reduces the marginal product of capital. This further decreases r^*, which reduces more than one-for-one with depreciation. When $\sigma < 1$ this effect is reversed and the decline is less than one-for-one, though still negative. This demonstrates the importance of the EIS for the effect of depreciation on the return on assets. This effect will also be present in the more general version of the model with multiple periods below.

When $\mu > 1$ there is an additional effect from the reallocation of savings toward equity. Equation (19) can be rewritten as

$$\hat{s}(r^*)\frac{1 - \alpha}{\mu} - \frac{(1 + g_T)}{r^* - g_T}\frac{\mu - 1}{\mu} = \frac{k^*}{y^*}(1 + g_T), \tag{21}$$

which shows that physical capital equals savings of the young less the value of equity. Consider $\sigma = 1$, such that savings of the young do not depend on r^*: An increase in depreciation lowers r^*, which increases the value of equity from Eq. (17). Consequently, when the young use their savings to purchase assets from the old, the higher price of equity leaves less for physical capital and therefore investments. Equilibrium y^*/k^* rises and compensates for

[18] These comparative statics are done for given g_T, and consequently an increase in g_Q would have to be combined with a decline in g_Z.

some of the decrease in r^*. This shifts of savings between physical capital and equity is important for the quantitative assessment of rising depreciation on r^* below.

Before proceeding, we discuss the uniqueness of the steady state equilibrium (naturally, there is always a steady state with $y^* = k^* = 0$). When the EIS is less than one—as in our preferred specification—the possibility of multiple steady states potentially arises (Galor, 1992). We check in our quantitative assessment below that the steady state is unique for our estimated set of parameters.[19]

However, with the existence of equity, even if the aggregate supply of savings depends negatively on r^*, the redistribution of savings away from equity (when r^* rises) could still leave savings for physical capital higher. This condition is easily met in our quantitative application in the following.

In the following section we distinguish between these findings and those of a Ramsey model.

4.3.2 Ramsey versus OLG

A central point of the present paper is the distinction between these results and those of a model with a representative agent. Therefore, compare the results in Proposition 1 with those of a model with a representative agent (as in FG). On the risky balanced growth path of a Ramsey model, the risky rate is given by:

$$1 + r^* = \beta^{-1} \left(\frac{1 + g_T}{1 + g_L} \right)^{\sigma} \left[E e^{(1-\theta)\chi_{t+1}} \right]^{\frac{\sigma-1}{1-\theta}}, \tag{22}$$

which is the familiar Euler equation with growth in consumption of the representative agent of $(1 + g_T)/(1 + g_L)$—the expected growth rate of the economy corrected for population growth—with an additional term that captures uncertainty.[20]

Consequently, the model with the representative agent leaves no room for depreciation in the determination of the return on capital, the supply curve of capital is entirely horizontal and the stock of capital adjusts to ensure the same return net of depreciation. This is illustrates in Fig. 4, which shows the equilibrium market for physical capital. In Panel a, we plot the equilibrium from the Ramsey model. The demand for capital (k^*/y^*) as a function of r^* is given Eq. (20) and the supply of physical capital comes from Eq. (22) which does not depend on k^*/y^*. The interest rate is given entirely by the supply curve and does not depend on δ: An increase in δ shifts the demand curve, reduces k^*/y^* and leaves r^* remains constant. Panel b shows the corresponding equilibrium for the OLG model. The demand for physical capital is the same, but the supply curve is given Eq. (19) which has a slope of:

[19] The issue of multiple steady states arises when the slope of the supply curve of capital is negative and therefore potentially has multiple intersections with the demand curve for capital. The existence of markups generally increases the slope of the supply curve and in our quantitative assessment the supply curve is always upward sloping. Therefore, no multiple steady states arise.

[20] Equation (22) can also be derived from the stochastic discount factor. For the representative agent, the analogous calculations as for Eq. (8) can be used to find:

$$M_{t+1} = \beta \left(\frac{1 + g_T}{1 + g_L} \right)^{-\sigma} e^{-\theta \chi_{t+1}} \left[E e^{(1-\theta)\chi_{t+1}} \right]^{\frac{\theta-\sigma}{1-\theta}}.$$

Using this to price the asset with return $(1 + r^*)e^{\chi_{t+1}}$ as $1 = E_t\left[M_{t+1}(1 + r^*)e^{\chi_{t+1}}\right]$ we recover Eq. (22).

$$\frac{dr^*}{d(k^*/y^*)} = \frac{1+g_T}{\hat{s}'(r^*)\frac{1-\alpha}{\mu} + \frac{(1+g_T)}{(r^*-g_T)^2}\frac{\mu-1}{\mu}}. \tag{23}$$

The slope of this curve is generally ambiguous. With $\sigma = 1$ and $\mu = 1$ it is a vertical line at a specific k^*/y^*. For $\mu > 1$ and $\hat{s}'(r)$ not "too" negative it will be upward-sloping as shown in the figure. The figure further shows the effect of an increase in δ which will now affect r^*. If the equity effect is large or the response of savings to the interest rate, $s'(r^*)$ are large the curve is flatter and a decline in r^* lowers k^*/y^* which dampens the effect of a decline in δ on r^*.

The present paper makes a sharp distinction between savings for life-cycle motives and the representative agent framework. In practice, bequests between generations could to some extent bridge the gab between the two. In Appendix 1 we extend a simplified version of this model (with no risk or monopoly power) to include a bequest motive. We show that in general a higher depreciation will continue to reduce r^*, although to a less extent depending on the strength of the bequest motive.

Having established the qualitative implications of an OLG framework, we proceed to quantitatively estimate our model. We will keep most of the structure but replace $\hat{s}(r^*)$ with the general savings function of an OLG model with longer life spans.

5 A multi-period OLG model and quantitative assessment

In the following, we quantitatively assess the importance of increased depreciation. To contrast the role of the overlapping-generation framework with that of a representative agent, we keep our empirical approach deliberately close to that of FG. They use the periods 1984–2000 and 2001–2016 and use 9 moments of data to estimate the model, separately for each time period, to exactly identify 9 parameters: (a) the risk price $\beta^* = \frac{1}{1+r^*}$, (b) risk modeled as a disaster γ,[21] (c) the markup μ, (d) the physical depreciation of capital δ, (e) the Cobb–Douglas parameter, α, (f)–(h) the growth rates of total factor productivity, g_Z, investment-specific progress, g_Q, and population growth g_L and (i) the labor force participation rate. The matched moments are: (i) measured gross profitability (including return on capital),[22] (ii) the measured gross profitability, (iii) the investment-capital ratio, (iv) The risk free rate, (v) the price-dividend ratio, (vi)–(viii) the growth rates of population, TFP

[21] FG model risk as a three-point distribution of χ_{t+1} with (a) a disaster: $\chi_{t+1} = 0$ with probability $(1-2\gamma)$, (b) $x_{t+1} = log(1-b)$ with probability γ and $c)$ $\chi_{t+1} = log(1+b_H)$ with probability γ. They set $e^b = 0.85$ estimate γ in the model and set b_H such that $Ee^{\chi_{t+1}} = 1$. We follow this approach, but the main conclusions of our analysis do not depend on this particular choice of a risk profile. (p denote the probability in their paper, but we use γ to avoid confusion with the value of equity, p^*)

[22] FG build their model slightly differently, in that they let firms own the capital stock such that gross profitability includes pure profits and return to capital. In such a model the only asset available for savings would be firm equity (which is then a claim on both monopoly rents and physical capital). Tobin's Q would give

$$p^*/k^* = (1+g_T)\left(1 + \frac{\mu-1}{\alpha}\frac{r^*+\delta+g_Q}{r^*-g_T}\right),$$

which is equation (24) in their paper. This would be the only asset to invest in such that in equilibrium $\hat{s}(r^*)\frac{1-\alpha}{\mu}y^* = p^*$ which gives the same equilibrium as in Proposition 1.

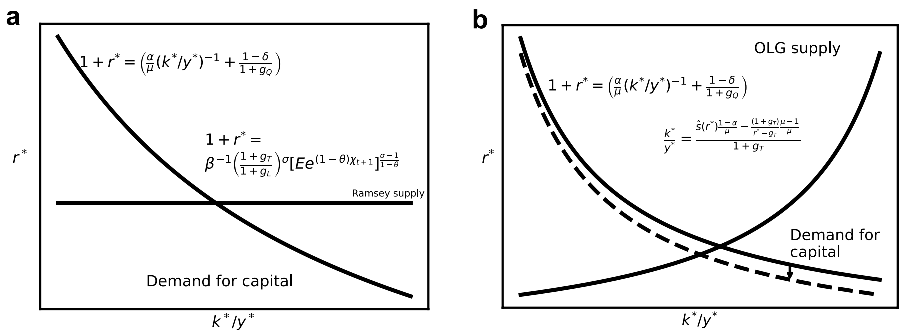


Fig. 4 Equilibrium in the market for physical capital *Note*: Figure shows the equilibrium in the market for physical capital. In both figures the demand for physical capital is given by gross return on capital less economic and physical depreciation. In the Ramsey model, the supply of physical capital is horizontal and given by the Euler equation of the representative agent. In the OLG model, the slope of the supply of physical capital comes from the savings of the young less the value of equity. In general the slope is ambiguous and given by Eq. (23)

and investment prices, and (iix) the labor force participation. They then discuss how variation in parameters drives the change in the rates of return.

Our approach differs from FG in three ways: First, we exclude their ninth moment: employment/population and, correspondingly, their parameter to capture this. Second, we focus on their first period 1984–2000 and perturb the rate of depreciation directly based on the change for US data from Sect. 2.2. Third, we estimate $\beta^* = \frac{1}{1+r^*}$ instead of β.[23] In the representative agent setting, neither of these changes significantly affect their conclusions (see Appendix 1). We will therefore also exactly identify our eight parameters. What is essential for our purposes is that the identification of these eight parameters is identical in the two models. The only difference between the two is how the equilibrium return β^* is tied to the fundamental parameter β.

In a representative agent model the relation between β and β^* comes from the stochastic discount factor of the representative agent (Eq. 23):

$$\beta^* = \frac{1}{1+r^*} = \beta\left(\frac{1+g_T}{1+g_L}\right)^{-\sigma}\left[Ee^{(1-\theta)\chi_{t+1}}\right]^{\frac{1-\sigma}{1-\theta}}, \tag{24}$$

whereas in the overlapping-generation model the link is given from combining our Eqs. (19) and (20):

$$\frac{\mu}{\alpha}\left(1 + r^* - \frac{1-\delta}{Q_{t+1}/Q_t}\right) = \frac{(1+g_T)}{\left\{\Lambda(r^*,\beta)\frac{1-\alpha}{\mu} - \frac{(1+g_T)}{(r^*-g_T)}\frac{\mu-1}{\mu}\right\}}, \tag{25}$$

where $\Lambda(r^*, \beta)$ is the generalization of $\hat{s}(r^*, \beta)$ to more generations (derived below). We make explicit the dependency on both r^* and β (though naturally both $\hat{s}$ and Λ depend on a broader set of parameters). The left hand side is the required y^*/k^* level consistent with an expected return of r^* and the RHS is y^*/k^* level arising from the savings behavior.

[23] The code from FG available on their website conveniently allows for the estimation of β^* instead of β.

Use Eq. (24) to define the function $f^{RA}(\beta, r^*) \equiv \beta\left(\frac{1+g_T}{1+g_L}\right)^{-\sigma}\left[Ee^{(1-\theta)\chi_{t+1}}\right]^{\frac{1-\sigma}{1-\theta}} - \frac{1}{1+r^*}$ and note that $f^{RA}(\beta, r^*) = 0$ defines β for any given set of parameters and r^*. Similarly, use Eq. (25) to define $f^{OLG}(\beta, r^*, \delta^{TOT}) = 0$, which defines β for the model with overlapping generations. Importantly, f^{OLG} is a function of δ^{TOT}, whereas f^{RA} is not (again, holding the growth rate of the economy constant).

In the OLG model, the effect of δ^{TOT} on the the real interest rate (holding the fundamental parameter β constant), is given by:

$$\frac{\partial r^*}{\partial \delta^{TOT}} = \frac{f^{OLG}_{\delta^{TOT}}}{f^{OLG}_{r^*}},$$

where $f^{OLG}_{\delta^{TOT}}$ denotes the derivative of f^{OLG} wrt δ^{TOT} and analogously for r^*.

In order to take this expression to the data, we note a practical problem: whereas the representative agent model of FG is estimated on yearly data, our specification of the OLG model features a lifespan of two periods, and consequently substantially longer periods. We therefore extend our model to multiple periods.

5.1 Extending the model to multiple periods

In the following, we allow our overlapping generations model to include multiple periods. In particular, we let an individual lifespan be T periods (years) out of which an individual works for the first G periods. We abstract from individual wage profiles and assume that any individual who works at time t earns the wage w_t. The work force is L_t and continues to grow at g_L. $L_{t,k}$ is the size of the workforce aged k at time t, such that $L_t = \sum_{k=1}^{G} L_{k,t}$. We leave the retirement age of an individual, G, exogenous, though endogenizing this would be an interesting extension.[24]

The solution to this model inherits the stable properties of the risky balanced growth path. In particular, in the appendix we show that:

Proposition 2 *Extend the overlapping generation model such that individuals live for T periods and earn income for the first G periods. All working individuals earn w_t. Then*

(a) *There exists an equilibrium with a risky balanced growth path where total wages, savings and output are proportional to $T_t S_t$ Individual wages grow at $(1+g) = (1+g_T)/(1+g_L)$.*
(b) *Let $W_{t,k}$ be the financial wealth of an individual born at time t of age k (such that $W_{t,1} = 0$ for a "new" individual with $k = 1$). Let $\hat{W}_{t,k}$ be total wealth, including future discounted wage income (including the current period t):*

[24] Carvalho et al. (2016) analyze the effect of the changing demographics, in particular longer lifespan, on the rate of return. They do so in an OLG model with young and old, but a stochastic transition from young to old and from old to dead for individuals. Longevity is modeled as a lower probability of dying. They find that increased savings from longer lifespans significantly affected the rate of return. We view the two OLG modeling approaches as complementary.

$$\hat{W}_{t,k} = W_{t,k} + \sum_{m=1}^{G-k+1} w_t \left[\frac{(1+g)}{1+r^*} \right]^{m-1},$$

where r^* is the discount factor using the return on the risky asset and the individual only earns income if they are still working, $k \leq G$.

(c) *On a risky balanced growth path total wealth of an individual of age k is given by*

$$\hat{W}_{t,k} = T_t S_t L_t^{-1} \hat{w}_k^*,$$

and individuals consume a time-independent, but age-dependent share of remaining total wealth: $(1-\hat{s}_k)\hat{W}_{s,k}$, where $\hat{s}_k$ is function of the return on assets β^* and is given iteratively for $k \in \{1, \dots, T-1\}$ as:

$$\hat{s}_k(r^*) = \frac{1}{1 + (1-\beta)^{1/\sigma}\left(\hat{v}_{k+1}\right)^{-1/\sigma}(r^*)^{\frac{\sigma-1}{\sigma}}\beta^{-1/\sigma}E\left(e^{(1-\theta)\chi_{t+1}}\right)^{\frac{\sigma-1}{\sigma}}},$$

$$\hat{v}_k(r^*) = \left((1-\beta)\left(1-\hat{s}_k\right)^{1-\sigma} + \hat{v}_{k+1}(r^*)^{1-\sigma}\left(\hat{s}_k\right)^{1-\sigma}\beta E\left(e^{(1-\theta)\chi_{t+1}}\right)^{\frac{1-\sigma}{1-\theta}} \right),$$

with $\hat{v}_T = (1-\beta)$ and $\hat{s}_T = 0$. For $\sigma = 1$, $\hat{s}_k$ is independent of r^*.

(d) *The financial wealth of an individual aged k at time t is given by*:

$$W_{t,k} = \lambda_k(r^*) w_t,$$

where $\lambda_k(r^*)$, the ratio of financial wealth of an individual aged k to current wages is independent of time.

(e) *Total savings in the economy at time t are proportional to total wages and are given by*:

$$\sum_{k=1}^{T} \lambda_k(r^*) L_{t,k} w_t = \frac{\sum_{k=1}^{T} \lambda_k(r^*) L_{t,k}}{\sum_{k=1}^{G} L_{t,k}} \sum_{k=1}^{G} L_{t,k} w_t \equiv \Lambda(r^*) \frac{(1-\alpha)}{\mu} Y_t,$$

where $L_{t,k}$ is the size of the population aged k at time t and Λ is not a function of t.

Proof See Appendix 1 □

The proposition extends the savings function of Eq. (5) and replicates the result from Sect. 4 when $T = 2$ and $G = 1$. Though the expression is explicit (see the appendix), evaluating it algebraically is complicated, and we solve for it numerically using the calibrated parameters below. In particular, based on a start of work life at age 20 we set $T = 65$ and working life $G = 40$.[25]

In Fig. 5 we plot the savings—that is without net present value of future wages—of a cross section of individuals and the increase in the expected return on assets from 6.6 to

[25] Given a starting life of 20, the value of T at 65 might seem high. Although the quantitative results of the effect of depreciation on the risk-free rate below depend only little on this assumption, a lower T gives too little aggregate wealth in the economy to match the data without a very high β (above 1). Any extension with bequests, uncertainty about time of death, housing, or direct utility of wealth would also increase savings.

7.6%.[26] Λ is the sum of generational savings where the weight is the average size of each generation. With no population growth, Λ is the integral under the plot. We show this figure for a range of values for the EIS ($1/\sigma$).

Panel *a* shows a high value of the EIS ($\sigma = 1/2$). The panel shows initial financial wealth of 0 with the generations at age 40 holding the highest stock of wealth. After retirement, savings decline and reach 0 at age 65. The figure is scaled by wage earnings (it shows λ_k) of a working individual such that an individual at age 40 has savings of around 8 times annual earnings for $r^* = 6.6\%$ (the estimated value below). We consider a 1 percentage point increase in the expected return. The decrease in the relative price of future consumption, pushes consumption towards the future and increases savings. With a high value of EIS this effect is large and the savings profile across generations increases substantially. Panel b plots the same figure, but for an EIS considerably lower than 1 ($\sigma = 3$). The substitution is now towards current consumption, but there is a contrary effect: the future discounted value of future labor income declines with a higher return which lowers current consumption. For these two panels these effects almost balance, and aggregate savings only increase a small amount. In panel c with $\sigma = 4$ there is a small decline in aggregate savings.

With aggregate savings Λ in hand we proceed to find the effect of an increase in depreciation on the risk-free rate.

5.2 The effect of depreciation on the risk free rate

We proceed to run the estimation procedure of FG. They consider two time periods 1984−2000 and 2001−2016. The procedure considers the economy to be in steady state in each of these periods and compares the change in parameters between the two periods. We base our parameter estimates on the period 1984−2000 (the first period of FG) since most of the increase in the depreciation rate happened in this period.[27] Since we do not calibrate a labor force participation rate, we have 8 parameters to which we add an equation to determine β. This is an updated version of Eq. (25), with Λ replacing $\hat{s}$ to allow for multiple generations:

$$\left(1 + r^* - \frac{1-\delta}{1+g_Q}\right)\frac{\mu}{\alpha} = \frac{1+g_T}{\left\{\Lambda(r^*,\beta)\frac{(1-\alpha)}{\mu} - \frac{(1+g_T)}{(r^*-g_T)}\frac{\mu-1}{\mu}\right\}}. \tag{26}$$

Table 1 shows the values of the corresponding parameters, including the estimate of β. β^* is estimated indepedently of σ, and we show three values of β for different values of σ. (note that δ, g_L, g_Z and g_Q have all been scaled by 100).

We use Eq. (26) to estimate the effect of a change in δ^{TOT} on r^*. We do so for a range of σ values. That is, for each value of σ we calculate the implied β and differentiate equation

[26] Note, this does not equal the savings profile of a given individual throughout her lifetime. This is so for two reasons: (i) people of older generations will have accumulated financial savings out of past (lower) wages, and (ii) this figure shows total savings for each generation, and correspondingly younger generations are bigger.

[27] Admittedly, there is an uncomfortable tension between Fig. 2 which shows a sizable increase in the rate of deprecation over the time period 1984−2000 and the estimation procedure which considers these parameters to be constant. Solving the model out of the steady state would be considerably more complicated. We believe that quantitatively similar effects would still be in play.

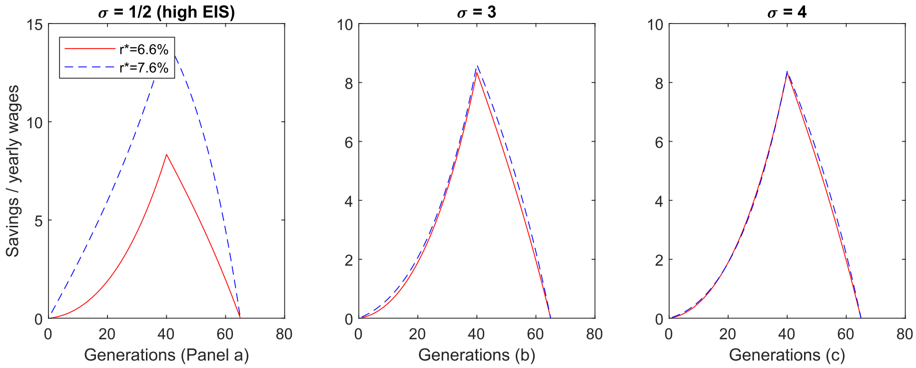


Fig. 5 Accumulated financial savings *Note*: Figure shows a cross-section of financial savings for each generation relative to yearly earnings of currently working person. Each panel presents a different elasticity of intertemporal elasticity and the effect on savings from an increase in the return on the risky asset, r^*

(26) with respect to δ^{TOT} holding β constant. The results are shown in Fig. 6. The intuition for the effect remains the same as in Proposition 1.

To illustrate this we consider both the case of a markup as estimated from the data, $\mu = 1.08$ and the corresponding model without markups ($\mu = 1$).[28] For each case we consider a range of σ. Panel *a* shows the results for $\mu = 1.08$ with all other parameters as in Table 1. We see that the extended OLG model replicates the qualitative effects of an increase in depreciation and continues to show substantial effects from increased depreciation on the return on capital. However, this effect strongly depends on σ for the same reason as illustrated in Fig. 4: A high EIS makes savings more sensitive to changes in r^* and creates a flatter supply curve of physical capital. For the estimated value of $\mu = 1.08$ and a value of $\sigma = 4$ the decline in deprecation of around 5/4 from 1970 to the early 2000s can explain around $\frac{5}{4} \times \frac{1}{4} = \frac{5}{16}$, around 30 points the decline in the real return on capital rate. If $\sigma = 1/2$ savings of young people are considerably more sensitive to r^*—the supply curve analogous to Fig. 4b for multiple generations is much flatter—and the effect of δ on $r^*$is negligible, around 7 points.

Panel *b* demonstrates the quantitative dependence on μ, the markup that allows for equity. When $\mu = 1$, savings are only invested in physical capital and the implied effect on the return to capital is around 90 points for $\sigma = 4$.

Finally, we use Eq. (9) and take logs with the approximation that $log(1 + r^f) \approx r^f$ to find an expression for the risk-free rate to get:

$$r^f \approx r^* - log\left[\frac{E_t e^{-\theta \chi_{t+1}}}{E_t(e^{(1-\theta)\chi_{t+1}})}\right], \tag{27}$$

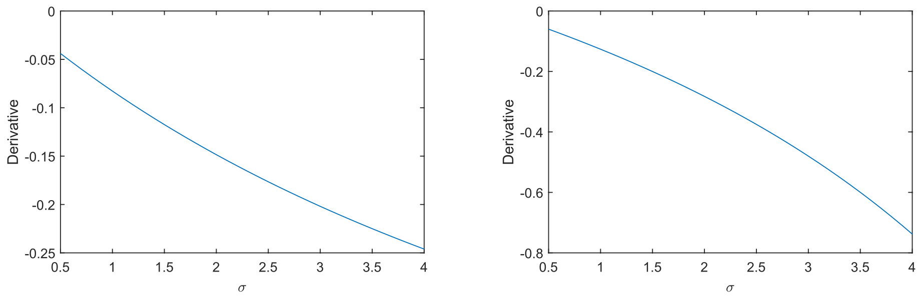
where $log\left[\frac{E_t e^{-\theta \chi_{t+1}}}{E_t(e^{(1-\theta)\chi_{t+1}})}\right] = 0.03$ with our set of estimated parameters. Consequently, for given risk characteristics and preferences, the risk-free rate follows the return on equity, and Fig. 6 applies equally to the risk-free rate.

[28] We keep all other parameters constant when we set $\mu = 1$. Consequently, this set of parameters does not match the empirical moments, in particular the profit share.

Table 1 Estimated parameters for 1984–2000

$\beta(\sigma = 1/2)$	$\beta(\sigma = 3)$	$\beta(\sigma = 4)$	β^*	μ	γ	δ	α	g_L	g_Z	g_Q
0.95	.98	0.99	0.94	1.08	0.04	2.1	0.24	0.6	1.30	1.77

*The parameters δ, g_L, g_Z, and g_Q have been scaled by 100



(a) Markup: $\mu = 1.08$

(b) Markup: $\mu = 1$

Fig. 6 Derivative of risky return, r^*, wrt total depreciation δ^{TOT}

Of course, r^* is the marginal product of physical capital, which is not easily measured in the data. An easier object to measure is the average product of capital. Caballero et al. (2017) discuss trends in the risk free rate and the average return to capital. To relate to this literature, in the following we calculate the average return on capital in our model. We change our model and let firms own the stock of capital. As stated in footnote 23 this alternative model is isomorphic to our preferred specification. The sum of the average gross (including depreciation) product of capital + profits is given by:

$$\frac{\alpha y^*}{\mu k^*} + \frac{1-\mu}{\mu}\frac{y^*}{k^*} = \left(\frac{\alpha + 1 - \mu}{\mu}\right)\frac{y^*}{k^*},$$

but in the literature (i.e. Eggertsson et. al., 2021) the average product of capital is measured net of depreciation (ideally both physical and economic) as:

$$APK = \left(\frac{\alpha + 1 - \mu}{\mu}\right)\frac{y^*}{k^*} - \frac{g_Q + \delta}{1 + g_Q}.$$

In the steady state the ratio of y^*/k^* is given by Eq. (20) which we use to write an expression for the APK over the risk free rate as:

$$APK - r^f = \frac{1-\mu}{\alpha}\left(r^* + \frac{g_Q + \delta}{1 + g_Q}\right) + r^* - r^f.$$

Empirically, APK has remained relatively constant while r^f has declined. This framework allows the spread between the two to arise from either monopoly rents or the risk premium.[29] depreciation $(q_Q + \delta)/(1 + g_Q)$ only affects the spread in that it increases y^*/k^*. Both Eggertsson et al. (2021) and Caballero et al. (2017) as well as a host of other papers find that increasing profits have played an important role in keeping *APK* constant despite the decline in risk free rate.

We have shown that the deviations of the Ramsey model from the OLG model depend strongly on the EIS: with a high value, savings are very sensitive to changes in the interest rate, and the OLG quantitatively mirrors the Ramsey model. With a low EIS, the supply curve is steeper, and changes in the demand for physical capital can have substantial effects on the return on capital and with it the risk free rate.

5.2.1 Empirical estimates of the elasticity of intertemporal substitution

There is a vast literature on EIS estimates. Havránek (2015) surveys 169 published studies and finds a mean estimate of around 0.5 ($\sigma = 2$). He argues that there is substantial selective reporting as scholars discard negative or insignificant results. When he corrects for this, he finds a mean estimate of micro studies of 0.3–0.4 with essentially zero from macro studies. Yogo (2004) argues that studies of the EIS based on the Euler equation suffer from weak instruments. In a macro study of 11 countries, he finds an EIS less than 1 and usually not different from zero. Best et al. (2020) use the particular structure of the UK mortgage market which features discrete jumps and employ methods from public finance on “bunching”. They find an EIS of around 0.1.

Taken together, these studies suggest that aggregate savings behavior and consumption are not highly sensitive to the interest rate, consistent with an upward-sloping supply curve in the OLG model. The literature suggests an EIS less than 0.5 and, correspondingly, a $\sigma > 2$.

6 Conclusion

Judged from the best available evidence, the average rate of capital depreciation has increased over the last half-century in the US and it seems likely that this trend is pervasive across advanced economies. A decomposition analysis finds that a rising share of capital assets featuring relatively high depreciation, such as IT, is chiefly responsible. This development is pervasive across sectors.

From a theoretical perspective a rising depreciation rate, whether driven by rising physical decay or revaluation, will work to lower the real rate of return in the steady state if aggregate savings are only modestly affected by changes in capital income. This amounts to saying that if the life-cycle motive is sufficiently important in explaining aggregate savings a rising depreciation rate should work to lower the real rate in the long run. This is in contrast to a model with a representative agent, where the Euler equation pins down the return. Our introduction of an overlapping-generation framework is relatively malleable. In this paper we focus on the real interest rate and rates of depreciation, but our framework could be used to address other research questions where the use of a representative agent might be too restrictive.

[29] Caballero et al. (2017) have a similar expression but consider a case where measured APK is only net of physical depreciation. In this case a higher rate of economic depreciation can also increase the spread between the two. Naturally, if *APK* does not adequately account for depreciation, a rising depreciation can also increase the spread.

Although the relative importance of the life cycle for savings remains an active area of research, our calibration suggests that the highlighted mechanism may have contributed significantly to the reduction in the risk-free rate since 1970, of around 30 basis points. In addition to contributing to the secular decline in the real rate, a rising depreciation rate could also contribute to a greater spread between the calibrated marginal (or average) product of capital and the risk-free real rate of interest.

Capital depreciation reflects both physical decay and revaluation. The revaluation element, in turn, reflects that declining investment prices represent a capital loss for capital owners, which serves to lower the return *ceteris paribus*. The present study can thus be seen as adding a complementary reason why declining investment prices may lower the real rate of return beyond that which has been explored in the literature. Declining investment prices may thus be considerable more important in explaining developments with respect to the real rate, in totality, than hitherto recognized. If productivity growth in the production of investment goods is expected to be above that in the production of consumption goods in the future, a declining investment price may serve to keep the real rate of return low in the future. Conversely, if the relative price of investment goods stabilizes, such that the rate of relative price decline is smaller, our model would predict a future *increase* in the rate of return.

7 Data repository

All code for the empirical figures and the simulation are available at https://github.com/mgolsen/real_rate_depreciation.

Appendix

A Data appendix

The following describes in further details the data behind the average depreciation rates. It is based on the current-cost net stock of assets and the real-cost net stock of assets (see page M-7 in U.S. Department of Commerce. Bureau of Economic Anayasis, 2003). The real stock of assets of type i in year t is given by $K^R_{i,t}$. This is measured in prices of a reference year, in this case 2017. The BEA then calculates the current cost value of the assets by $K^N_{i,t} = P_{i,t} \times K^R_{i,t}$, where $P_{i,t}$ is the corresponding price index. $K^N_{i,t}$ then measures the price of the stock at time t and not in 2017. The depreciation rate for each capital type, $\delta_{i,t}$, can be found by taking total depreciation and dividing by the stock of assets. One can then calculate the average depreciation rate—by nominal and real weights, respectively—as:

$$\delta_t^N = \frac{\sum_i K_{i,t}^N \delta_{i,t}}{\sum_i K_{i,t}^N},$$

$$\delta_t^R = \frac{\sum_i K_{i,t}^R \delta_{i,t}}{\sum_i K_{i,t}^R}$$

$$= \frac{\sum_i \left(K_{i,t}^N / P_{i,t}\right)\delta_{i,t}}{\sum_i \left(K_{i,t}^R / P_{i,t}\right)},$$

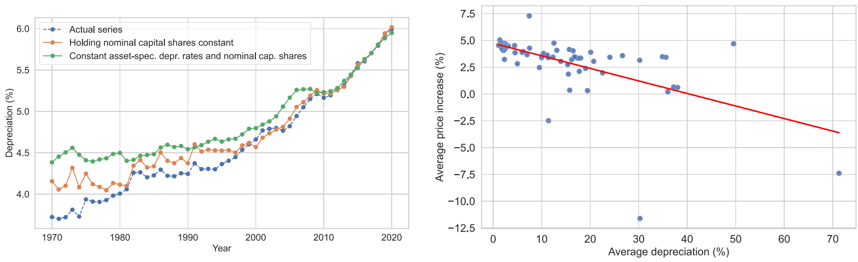
such that δ_t^N can change due to reallocation of $K_{i,t}^N$ across asset types or $\delta_{i,t}$ (as shown in Panel b of Fig. 3). δ_t^R can also change due to different paths in $P_{i,t}$. As argued in the main body of the paper, δ_t^N is preferable. To illustrate this, consider panel 7a. This shows three series, all of which intersect in 2017, the baseline year for real stock calculations. First, it shows the depreciation rate weighted by the real stock of value (Actual series). This corresponds to Panel b of Fig. 1, though using more detailed BEA data instead of the PWT. To illustrate the problem with using real values, consider a hypothetical economy in steady state: There is no change in the nominal share of different equipment types and the depreciation rate for each equipment type is held constant. There is, however, different growth rates for the price index of different capital types. The depreciation rate from the perspective of agents is constant, but using real capital to measure the depreciation rate will enduce a bias in the measure.

We create two counterfactual data series. First, we consider a hypothetical series in which the weights of nominal capital $K_{i,t}^N / \sum_i K_{i,t}^N$ is constant at its 2017 level throughout the period; changes in δ^R can come only from changes in asset-specific depreciation or different growth rates in price indices. The figure shows this alternative series as "Holding nominal capital shares constant". Since nominal asset growth has been the highest for assets with a high depreciation rate, this implies a higher depreciation rate in 1970; an increase from the actual series of about .5 points. Second, we consider a counterfactual world where both the relative weight of nominal capital and the depreciation rate are held constant at 2017 values, such that only the effect of $P_{i,t}$ is present. For this series, the depreciation rate weighted by real capital grows by around 1.5 points from 1970 to 2017. This illustrates that in a counterfactual world with no change in relative nominal assets values or asset-specific depreciation rates, the average depreciation rate would still show growth. Due to the approximate exponential feature of price indices, this series will also appear exponential.

Panel b shows a scatter plot across the different asset types. It shows the average yearly price change for each asset plotted against the (arithmetic) average of depreciation across the time period (the very high depreciation rates are found within software). There is a clear negative relationship between the two: assets with the highest depreciation rates have seen the highest declines in prices and consequently have artificially little weight early in the time period when using the real stock of assets.

B A model with a bequest motive

We present a simple version of our 2-generation model, where we allow for bequests. We abstract from monopoly rents and uncertainty and focus on logarithmic preferences. We also let all generations have the same size, $L_t = 1$



(a) Average depreciation weighted by real assets (b) Average depreciation rate and average price change (1970-2020)

Fig. 7 The rate of depreciation and the real stock of assets (1970–2020). *Note*: Panel a shows the average depreciation rate weighted by the real stock of assets. It performs two counter-factuals: Holding the nominal share of assets constant at 2017 values. Holding both nominal shares and depreciation rates constant for each asset type. Panel b shows the average depreciation rate against the average price decline (23)

B.1 Consumers

Individuals live for two periods. In the first period of life, they supply a unit of labor inelastically for which they receive a wage. They receive bequests and save. During old-age individuals consume and pass on bequests to their heir from their savings during youth. Lifetime utility for a generation born at time t is

$$u_t = \ln c_{1,t} + \frac{1}{1+\rho}(\ln c_{2t+1} + \eta \ln b_{t+1}).$$

where $c_{1,t}$ is consumption during youth and $c_{2,t}$ is consumption during retirement, b_{t+1} is bequests while ρ represents time preferences. Per period utility is logarithmic, which implies that the savings rate will be independent of the real rate; we discuss below how our results are likely affected if this assumption is relaxed.

The bequest motive is captured by joy-of-giving preferences, and η parameterizes the strength of the bequest motive. An alternative modeling approach is to assume that households behave dynastically, deriving utility from the utility of the descendants and so on. But this will produce a model where the aggregate savings behavior is isomorphic to an infinitely lived consumer if bequests are passed on, which implies that steady-state savings solely depend on capital income (Barro, 1974). In order to produce behavior where both wages and capital income matter, we, therefore, resort to the joy-of-giving specification.[30]

The budget constraints are

$$c_{1t} + s_t = w_t + b_t \equiv I_t,$$
$$(1 + r_{t+1})s_t = c_{2t+1} + b_{t+1},$$

where the assumption that each generation is of equal size eliminates any need to normalize first period bequest. Accordingly, the first period income comprises wage income, w_t, and bequest, b_t. Second period income consists of savings during youth with interest, where r_{t+1} is the real rate of interest.

[30] The assumption that consumer welfare depends on terminal wealth, or bequest, goes back to Yaari (1964). The empirical relevance of a bequest motive is well established, albeit the strength of the motive, relative to the life-cycle motive, is an active area of research. See De Nardi et al. (2016).

Maximizing life-time utility subject to these constraints leads to the following solutions for first period savings and late-in-life bequest

$$s_t = \frac{1+\eta}{2+\rho+\eta} I_t,$$
$$b_{t+1} = \frac{\eta}{1+\eta}\left(1+r_{t+1}\right)s_t.$$

Log preferences imply that the savings rate is independent of the real rate of return. The presence of a bequest motive ($\eta > 0$) increases the savings rate compared to a standard two-sector OLG model. In addition, consumers divide their accumulated lifetime wealth between their own consumption in old age and the bequest of their offspring; the split becomes more favorable to the next generation if η increases in size.

Taken together, this behavior implies, in contrast to a our baseline model with logarithmic preferences, that the real rate of return will influence the process of capital accumulation. A higher real rate increases accumulated savings, which translates into greater bequests and thus the income of the next generation, which then fuels more savings. The strength of this channel depends on the size of η. As a result, both wage income and capital income will influence capital accumulation in the present setting.

B.2 Production

Production continues to have the same structure as in the baseline model. However, with no monopoly profits and no risk the accumulation of capital is simpler:

$$Q_t^{-1} K_{t+1} = s_t,$$

where we recall that the size of generations has been normalized to one. Substituting for optimal savings as well as (lagged) bequest leads to

$$Q_t^{-1} K_{t+1} = \frac{1+\eta}{2+\rho+\eta}\left[\frac{\eta}{1+\eta}\left(1+r_t\right)Q_t^{-1}K_t + (1-\alpha)Y_t\right].$$

The terms in the square bracket reflect that the income of the young is partly based on capital income, via bequests, and partly on labor income. The two motives for savings, life-cycle and bequest, both influence capital accumulation, and their relative importance is determined by η; if $\eta = 0$ only wage income matters to the process of capital accumulation.

If we define k^* in an analogous manner we get a steady-state relationship as:

$$k^* = \frac{1}{1+g_T}\frac{1+\eta}{2+\rho+\eta}\left\{\left[\frac{\eta}{1+\eta}\alpha + (1-\alpha)\right]y^* + \frac{\eta}{1+\eta}\frac{(1-\delta)}{1+g_Q}\right\},$$

where $1 + g_T$ is the long-run growth rate of output and

$$y^* = (k^*)^\alpha \left(\frac{1}{1+g_Q}\right)^\alpha,$$

and we can show the

Table 2 Contribution of different parameters to the decline in the risk-free rate: FG original and FG with our changes

Estimation	r^* ('84-'00)	r^* ('01-'16)	Difference	β	β^*	μ	γ	δ	α	gN	gZ	gQ
FG	2.79	−0.35	−3.14	−1.22	n/a	0	−1.62	0	0	0	−0.19	−0.10
FG with our changes	2.79	−0.35	−3.14	n/a	−2.10	0	−1.04	0	0	0	0	0

$$\frac{y^*}{k^*} = \frac{(1+g)(2+\rho+\eta)}{\eta\alpha+(1+\eta)(1-\alpha)} - \frac{\eta\frac{(1-\delta)}{1+g_Q}}{\eta\alpha+(1+\eta)(1-\alpha)}, \tag{28}$$

In the steady state a declining price of investment goods serves to increase the average productivity of capital due to accelerating economic depreciation, capturing physical decay as well as revaluation. This only happens when the bequest motive is operative, as can be seen. The underlying cause is that when economic depreciation increases it reduces the return to savings, thus bequests and therefore the income of the young consumers. Capital accumulation is thereby reduced and this increases long-run average capital productivity.

The reason why relative prices appear nowhere else in the formula *except* via depreciation is due to the Cobb–Douglas production technology which features an elasticity of substitution between capital and labor of one. The two "traditional effects", discussed above, whereby a given amount of savings buys more capital—thus stimulating investments—and a declining marginal product of capital—thus discouraging investments—exactly cancel out.

We then recover the rate of return (risky or risk-free, equivalently):

$$1+r^* = \alpha\frac{y^*}{q^*} + \frac{(1-\delta)}{1+g_Q} = \frac{\alpha(1+g)(2+\rho+\eta)}{\eta\alpha+(1+\eta)(1-\alpha)} + \frac{(1+\eta)(1-\alpha)}{\eta\alpha+(1+\eta)(1-\alpha)}\frac{(1-\delta)}{1+g_Q}, \tag{29}$$

In contrast to the baseline model, where the effect with logarithmic preferences is one for one, the effect is now smaller, depending on the bequest motive. When $\eta = 0$ we recover the original result.

C Quantitative estimate of the contribution from each parameter in FG

We first run the FG estimate with our two changes: estimating β^* instead of β and without the distinction between workforce and employment. The results are given in Table 2. Though the exact estimates vary, both estimates show that changes to β (β^*) and γ, the probability of disaster, are the major contributors to the decline in the risk free rate.

C.1 Overlapping generations in multiple periods

The utility function for an individual of age s at time t is defined as:

$$V_{t,s} = \left((1-\beta)c_{t,s}^{1-\sigma} + \beta E\left(V_{t+1,s+1}^{1-\theta}\right)^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}$$

for $s = t, \ldots T-1$. In the last period of life the utility function is:

$$V_{t,T} = (1-\beta)^{\frac{1}{1-\sigma}} c_{t,T}.$$

Financial wealth follows the process:

$$W_{t+1,s+1} = \frac{e^{\chi_{t+1}}}{\beta^*}\left[W_{t,s} + 1_{s\le G}w_t - c_{t,s}\right],$$

since a unit of the risky asset costs β^* and therefore has return $e^{\chi_{t+1}}/\beta^*$ (on the risky balanced growth path).

Step 1. Optimal savings

To find the solution we consider a risky balanced growth path and start at the second to last stage of life.

Since the individual does not work the last period of life, consumption must be (from the perspective of the second-to-last period):

$$c_{t+1,T} = \frac{e^{\chi_{t+1}}}{\beta^*}\left(W_{t,T-1} + 1_{T-1\le G}w_t - c_{t,T-1}\right) \equiv \frac{e^{\chi_{t+1}}}{\beta^*}\left(\hat{W}_{t,T-1} - c_{t,T-1}\right),$$

which covers both the case where the individual works ($T-1 \le G$) and the one where the individual doesn't. $\hat{W}_{t,T-1}$ is consequently total wealth for the individual in the beginning of their $T-1$ period of life.

The maximization problem is then

$$max_{c_{t,T-1}}\left((1-\beta)c_{t,T-1}^{1-\sigma} + \beta(1-\beta)(\beta^*)^{\sigma-1}\left(\hat{W}_{t,T-1} - c_{t,T-1}\right)^{1-\sigma} E\left(e^{(1-\theta)\chi_{t+1}}\right)^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}},$$

The first order conditions for this returns savings of

$$\hat{W}_{t,T-1} - c_{t,T-1} = \frac{1}{1+\beta^{-1/\sigma}(\beta^*)^{\frac{1-\sigma}{\sigma}} E\left(e^{(1-\theta)\chi_{t+1}}\right)^{\frac{\sigma-1}{\sigma}\frac{1}{1-\theta}}}\hat{W}_{t,T-1} \equiv \hat{s}_{T-1}\hat{W}_{t,T-1},$$

with consumption of $(1-\hat{s}_{T-1})W_{t,T-1}$. This naturally replicates the savings expression of the two period model except savings is out of accumulated wealth and not first-period earnings.

This returns utility of:

$$V_{t,T-1} = \left((1-\beta)\left(1-\hat{s}_{T-1}\right)^{1-\sigma} + \beta(1-\beta)(\beta^*)^{\sigma-1}\left(\hat{s}_{T-1}\right)^{1-\sigma} E\left(e^{(1-\theta)\chi_{t+1}}\right)^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}} \hat{W}_{t,T-1} \equiv \hat{v}_{T-1}^{\frac{1}{1-\sigma}}\hat{W}_{t,T-1}.$$

Consequently, both savings and utility scale with total wealth.

The maximization problem in this period is:

$$max_{c_{t,T-2}}\left((1-\beta)c_{t,T-2}^{1-\sigma} + \beta E\left(V_{t+1,T-1}^{1-\theta}\right)^{\frac{1-\sigma}{1-\theta}}\right)^{\frac{1}{1-\sigma}}$$

where:

$$\begin{aligned} V_{t+1,T-1} &= \hat{v}_{T-1}^{\frac{1}{1-\sigma}}\left\{\frac{e^{\chi_{t+1}}}{\beta^*}\left(W_{t,T-2} + 1_{T-2\le G}w_t - c_{t,T-2}\right) + (1+g)e^{\chi_{t+1}}1_{T-1\le G}w_{t+1}\right\} \\ &= \hat{v}_{T-1}^{\frac{1}{1-\sigma}}\frac{e^{\chi_{t+1}}}{\beta^*}\left\{\hat{W}_{t,T-2} - c_{t,T-2}\right\}, \end{aligned}$$

which gives a first order condition of:

$$\hat{W}_{t,T-2} - c_{t,T-2} = \frac{1}{1+(1-\beta)^{1/\sigma}\hat{v}_{T-1}^{-\frac{1}{\sigma}}(\beta^*)^{\frac{1-\sigma}{\sigma}}\beta^{-1/\sigma}E_t(e^{(1-\theta)\chi_{t+1}})^{\frac{\sigma-1}{\sigma}\frac{1}{1-\theta}}}\hat{W}_{t,T-2} \equiv \hat{s}_{T-2}W_{t,T-2}$$

with

$$V_{t,T-2} = \left((1-\beta)\left(1-\hat{s}_{T-2}\right)^{1-\sigma} + \hat{v}_{T-1}(\beta^*)^{\sigma-1}\left(\hat{s}_{T-2}\right)^{1-\sigma}\beta E\left(e^{(1-\theta)\chi_{t+1}}\right)^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}} \hat{W}_{t,T-2} \equiv \hat{v}_{T-2}^{\frac{1}{1-\sigma}} \hat{W}_{t,T-2}.$$

Continuing this progress allows us define $\left\{\hat{s}_s, \hat{v}_s\right\}_{s=1}^{T}$ iteratively as:

$$\hat{s}_s = \frac{1}{1 + (1-\beta)^{1/\sigma}\hat{v}_{s+1}^{-1/\sigma}(\beta^*)^{\frac{1-\sigma}{\sigma}}\beta^{-1/\sigma}E_t(e^{(1-\theta)\chi_{t+1}})^{\frac{\sigma-1}{\sigma}\frac{1}{1-\theta}}},$$

$$\hat{v}_s = (1-\beta)\left(1-\hat{s}_s\right)^{1-\sigma} + \hat{v}_{s+1}(\beta^*)^{\sigma-1}\left(\hat{s}_s\right)^{1-\sigma}\beta E\left(e^{(1-\theta)\chi_{t+1}}\right)^{\frac{1-\sigma}{1-\theta}},$$

with $\hat{v}_T = (1-\beta)$, $\hat{s}_T = 0$ and:

$$\hat{W}_{t,s} = W_{t,s} + \sum_{k=s}^{T} 1_{k\le G} w_t(\beta^*(1+g))^{k-s},$$

where the second term is the future wage income discounted at the expected return on risky capital $1/\beta^*$. We call $\hat{W}_{t,s}$ total wealth.

With this in hand we proceed to find the wealth for each generation.

Step 2: Financial wealth

With these functions in hand we can find the financial wealth (or debt) at each point in time for each generation.

Consider first a person aged 1 at time t who has no initial financial savings. Their financial savings at the end of their first period of life are $s_{t,s}$:

$$s_{t,1} = w_t - (1-\hat{s}_1)\hat{W}_{t,1} = \left\{ 1 - (1-\hat{s}_1)\sum_{k=1}^{T} 1_{k\le G}(\beta^*(1+g))^{k-1} \right\} w_t = \lambda_1 w_t$$

Now consider a person aged 2 at time t. By the logic above they saved $s_{t,1}$ the previous period with stochastic return of e^{χ_t}/β^*. Consequently, their savings will be:

$$\begin{aligned} s_{t,2} &= w_t 1_{2\le G} + \frac{e^{\chi_t}}{\beta^*}\lambda_1 w_{t-1} - (1-\hat{s}_2)\hat{W}_{t,2} \\ &= w_t 1_{2\le G} + \frac{1}{\beta^*(1+g)}\lambda_1 w_t - (1-\hat{s}_2)\left\{ \frac{1}{(1+g)\beta^*}\lambda_1 w_t + \sum_{k=2}^{T} 1_{k\le G}(\beta^*(1+g))^{k-2} \right\} \\ &= w_t 1_{2\le G} + \frac{\hat{s}_2}{\beta^*(1+g)}\lambda_1 w_t - (1-\hat{s}_2)\left\{ \sum_{k=2}^{T} 1_{k\le G}(\beta^*(1+g))^{k-2} \right\} \\ &\equiv \lambda_2 w_t \end{aligned}$$

which consists of two parts: The financial savings from previous period (corrected for return and the lower wage in the previous period) + current income from which consumption in period t is subtracted. The ratio of current financial savings to current wage income is independent of time. We can iteratively find all $\left\{\lambda_k\right\}_{k=1}^{T}$ in the same manner.

In particular for period 3 we have:

$$s_{t,3} = w_t 1_{3\le G} + \frac{e^{\chi_t}}{\beta^*}\lambda_2 w_{t-1} - (1-\hat{s}_3)\hat{W}_{t,3}$$

Finally, to get total savings of the economy we need to correct for the growing population size. Consider a time period at which the size of the youngest population is $L_{t,1}$ and the corresponding size of older generations $L_{t,s} = (1 + g_L)^{-s+1}$. Consequently letting Λ be the share of wage income that is saved.

$$\Lambda = \frac{\left\{ \sum_{k=1}^{T} \lambda_k \times (1 + g_L)^{-k+1} \right\} w_t L_{t,1}}{w_t \sum_{k=1}^{G} (1 + g_L)^{-k+1} L_{t,1}} = \frac{\left\{ \sum_{k=1}^{T} \lambda_k \times (1 + g_L)^{-k+1} \right\}}{\sum_{k=1}^{G} (1 + g_L)^{-k+1},}$$

which is independent of t

Acknowledgements We would like to thank Jacob Gyntelberg, David Hémous, Holger Strulik, and Asger Wingender for useful comments and suggestions as well as Marcus Piil Pedersen and Emilie Vestergaard for excellent research assistance. The opinions expressed are those of the authors and do not necessarily reflect those of the Danish Economic Councils.

Funding Open access funding provided by Copenhagen University.

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