Has the real rate of return "depreciated"?*

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Abstract

We document that over the past half a century the average depreciation rate on capital has increased in the United States and most likely across the bloc of Advanced Economies. A decomposition analysis for the United States reveals that increasing capital depreciation is a within sector phenomenon. Essentially none of the run-up since 1970 can be attributed to reallocation. Exploring the economic consequences of rising capital depreciation, in the shape of either physical decay or a declining price of investment goods, we argue that it has reduced the steady-state real rate of return and contributed to a (greater) gap between the marginal product of capital and the real rate on risk free assets. These predictions depend on the relative strength of the bequest and life-cycle motives in generating savings. A plausible calibration indicates depreciation may have been a significant force behind the observed secular decline in the natural real rate of return over the last half century.

Keywords: The Real Rate of Interest; Capital depreciation; Growth theory; Savings motive
JEL Codes: E43, O4, E13, E21

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1 Introduction

Over the past fifty years, the natural real rate of interest in Advanced Economies has declined by about 300 bps (Rachel and Summers, 2019).[1] This has been accompanied by a general decline in the real rates on safe assets. At its most basic level, the observed decline can be attributed to a rise in aggregate savings, a decline in aggregate investments, or a combination of the two. But what perhaps makes the decline particularly challenging to explain lies in the fact that the marginal product of capital does not appear to have declined simultaneously; if anything it has increased slightly (cf. Gomme et al, 2011; Caballero et al., 2017; Eggertsson et al, 2021). We argue that a hitherto neglected factor may have contributed significantly to these developments: a rising rate of capital depreciation.

As documented below the average rate of capital depreciation appears to have risen by around one percentage point in the United States over the last half-century, and the best available cross-country data suggests that this run-up has occurred in general within the group of Advanced Economies. Using detailed US data from the Bureau of Economic Analysis we find that the rise in average depreciation is pervasive across sectors. Within the US inter-sectoral reallocation has essentially played no role in the increase in average depreciation. Empirically it is not possibe to say how much of the observed increase in depreciation is due to physical depreciation and how much is due to revaluation albeit intuition may perhaps suggest the latter is more important when it comes to IT-related assets. However, we show that a rising rate of depreciation may reduce the steady state real rate – while keeping the marginal product of capital constant (or induce it to rise) – regardless of whether it is due to physical wear and tear or revaluation.

To support this claim we construct a two-sector overlapping generations model of a closed competitive economy. The model is designed to capture three key mechanisms. First, we assume that (exogenous) technological change is faster in the investment goods sector than in the consumption goods sector. Otherwise, the production technologies of the two sectors are identical, Cobb-Douglas, and employ both capital and labor as inputs. These assumptions, along with competitive markets, imply that the (relative) price of investment is declining over time at a constant rate. Second, competitive markets also mean that the marginal product of capital equals the user-cost of capital at all points in time. The user-cost of capital depends on the cost of borrowing (i.e., the real rate of return), physical depreciation, and the rate of change in the relative price of investment. When the relative price of investment goods continuously declines (or the rate of physical decay goes up) the user-cost of capital increases. The reason why prices matter is because the act of investment involves a capital loss: the price at which the capital good is purchased is higher than the resale value after a period. Hence we capture faster depreciation as *either* greater physical decay *or* a faster rate of investment good deflation. Third, households simultaneously have a life-cycle motive as well as a bequest motive for saving, and both motives matter for aggregate savings. Utility is logarithmic which ensures that the savings *rate* of households is independent of the real interest rate. The substantive implication is that aggregate savings will depend on wage income and capital income and that savings through capital income derive from the presence of a bequest motive.[2]

These three elements interact in the following way. Consider an acceleration in the speed of techno-

[1] Rachel and Summers (2019) estimate the natural real rate of interest as the interest rate that is consistent with output at its potential structural level and constant inflation.

[2] We discuss below how results are affected if we relax the assumption of log utility.

logical change in the investment goods sector relative to the consumption goods sector. This leads to a faster rate of (relative) investment price deflation. To fix ideas suppose this experiment leaves the steady state growth rate of the economy, which is a weighted average of technological change in the two sectors, unaffected. When economic depreciation rises the instantaneous demand for capital declines since the user cost of capital rises. For the marginal product of capital given this lowers the real rate in the model.[3]

The fact that production technologies are identical and Cobb-Douglas ensures that a familiar mechanism, linking a *change* in the relative price of investment to the real rate, is not present: Holding the marginal product constant, cheaper investment goods means that a given amount of savings buys more investment, which increases the return to investment, but holding savings constant more investment lowers the marginal product of capital, which reduces the return on investments. The net effect on the real rate is generally ambiguous, but if the elasticity of substitution is smaller than one the real rate is reduced in equilibrium (e.g., Sajedi and Thwaites, 2016). With identical Cobb-Douglas technologies, these effects cancel out. Canceling the familiar effects of a change in the relative of investment allows us to focus on the effects from changes to the *rate* of decline in the price of investment goods. [4]

While there is a negative direct effect of rising depreciation on the equilibrium real rate, the ultimate impact of the experiment on the real rate depends on how capital accumulation is influenced by the shock. To see the intuition clearly, suppose for a moment that households do not have a bequest motive. In that case, aggregate savings only depend on wage income since the real rate does not affect the savings rate. Faster economic depreciation will therefore leave capital accumulation and thus the marginal product of capital unaffected in the steady state. Consequently, the steady state real rate declines and a gap emerges between the marginal product and the real rate.

Naturally, if the bequest motive is present things are slightly more complicated. The reason is that when the user-cost of capital increases it manifests in a lower return to savings which lowers aggregate bequest and thereby stifles capital accumulation. This produces a countervailing effect of depreciation on the steady state real rate as less long-run capital increases the marginal product of capital. As shown formally below the net effect on the real rate is still negative but the marginal product of capital rises. The intuition why the real rate declines on net, is that the direct effect from depreciation on the real rate is a first order effect whereas the positive effect on the real rate via accumulation is second order.

In general, the "pass through" of economic depreciation on the real rate depends on the relative strength of the bequest-and life-cycle motive for savings. As discussed in Section 3 below, different workhorse growth models in macroeconomics hold very different predictions for the depreciation/real rate nexus precisely because they assume different savings motives. In a Diamond (1965) model the life-cycle motive is the only savings motive, and a change in depreciation strongly affects the steady state real rate, whereas the bequest motive is all-important in a Ramsey-Cass-Koopmans model (Barro, 1974) where depreciation has no effect on the steady state real rate. In practise both life-cycle and bequests undoubtedly matter simultanously suggesting a long-run impact on the real rate from depreciation some-

[3]The steady state results from this experiment on the real rate and marginal product of capital and an experiment that involves faster *physical* depreciation are *identical* in the model, except that physical depreciation has no potential impact on the growth rate of the economy.

[4]While the mechanism discussed in Sajedi and Thwaites (2016) and the one in focus here are mutually reinforcing with respect to the real rate of interest only the depreciation channel produces a gap between the real rate and the marginal product. On the other hand, the mechanism explored by Sajedi and Thwaites links declining relative investment prices to the labor share. With the Cobb-Douglas assumption, the aggregate labor share does not move.

where in between the predictions of the Diamond and Ramsey-Cass-Koopmans model. We propose a calibration of the model that allows us to gauge what an empirically plausible impact might be. An increase in the depreciation rate of one percent (regardless of whether it is attributable to declining prices or greater physical depreciation) is found to lower the steady state real rate of return between 60 bps and 100 bps, with 75 basis points as the best guess. These are significant effects in light of the observed decline in the natural real rate of return (the natural empirical counterpart to a steady state real rate of return in a non-stochastic growth model) of about 300 bps.

Below we also discuss a historical challenge for several existing explanations for the recent (post-1970) decline in the risk-free rate of interest. The challenge lies in accounting for the apparent fact that a secular decline has been underway for much longer. Recent work by Schmelzing (2019) suggests that the period since 1820 has witnessed a negative trend of about 2.29 bps per year on average. Over time horizons as long as these, most theoretical determinants of the real rate receive mixed support (e.g., Borio et al., 2017; Lunsford and West, 2019). This raises the question of whether the "depreciation-mechanism" in focus here suffers from a similar "external validity" issue.[5]

Unfortunately, reliable data on economic depreciation is not available for this entire period. Hence, while it is possible that economic depreciation has increased, such that the mechanism in focus speaks to the regularity uncovered by Schmelzing, no quantitative data can be brought to bear. But an alternative line of attack is viable since a reasonable case can be made that the bequest motive is relatively *less* important to savings today, compared with the situation around the time of the Industrial Revolution. This matters as it implies a greater *pass through* from economic depreciation to the real rate, *ceteris paribus*. That is, even if depreciation stays constant over the long run the mechanism explored in this paper will nevertheless increasingly work to depress the real rate insofar as the bequest motive recedes in importance. We discuss plausible magnitudes and conclude that economic depreciation may help explain the long-run decline in the real rate albeit the quantitative significance of the mechanism is probably modest.

The paper is related to the recent literature that discusses plausible explanations for the observed decline in real rates of interest over the last half century. Several contributing forces have been put forward. Useful overviews of the literature are found in Rachel and Smith (2017), Rachel and Summers (2019), and Kiley (2020). Since the evolution of the relative price of investment is a key part of the analysis work connecting the relative price of investment to the real rate is related (e.g., Sajedi and Thwaites, 2016). As explained above, it is the rate of change that matters in the present paper rather than the level and we make assumption that ensure that the familiar mechanism from the literature is not present. But more broadly the contribution of the present paper is to explore the relevance of capital depreciation, a factor that seems neglected so far. Similarly related are contributions that aim to explain the gap between the marginal product and the real rate, e.g. via rising market power or rising risk premia; Eggertsson et al (2021) contains an overview. Rising depreciation is a mechanism left unexplored in this literature as well.[6]

[5]The work by Schmelzing suggests this negative trend can be observed since the 15th century. Here we limit attention to the period after the turn of the 19th century with an eye to the theoretical frameworks used below, which probably have limited relevance before the onset of modern growth. Back then the mechanics of the growth process were substantively different (e.g., Ashraf and Galor, 2011), which an account for a (very) long-run decline in the interest rate would need to take into account.

[6]An exception is Farhi and Gourio (2018) who do explore the importance of depreciation for gap between the marginal product and the risk free rate, concluding it has been unimportant. The reason is presumably that the analysis is conducted in a (stochastic) Ramsey-Cass-Koopmans model where depreciation does not matter to the steady state real rate. See the

Since the relative importance of bequest and life-cycle savings is key to the question at hand, the paper is indirectly related to the literature which has examined this question, though mainly from an empirical perspective (e.g., Kottlikoff and Summers, 1981; Modigliani, 1988; Dynan et al., 2002; Piketty, 2011). A central point made in the present paper is that establishing the relative importance of these two fundamental savings motives has important implications for the determinants of the long-run real rate of interest. Similarly related is therefore work which shows how the savings motive matters for a range of other key predictions that flow from standard growth theory. This includes Bertola (1996) on the link between the factor income distribution and growth; Galor (1996) on convergence properties (conditional vs. club convergence); Uhlig and Yanagawa (1996) on the impact of capital taxation; Jones and Manuelli (1990) on the viability of endogenous growth, and Dalgaard and Jensen (2009) on scale effects in endogenous growth models.

The paper proceeds as follows. In the next section, we document that the average rate of depreciation has gone up. Section 3 provides a brief review of standard neoclassical workhorse models concerning the nexus between depreciation and the steady-state real rate of return. Section 4 provides our two-sector OLG model where both life cycle savings and bequest are operative savings motives and influence capital accumulation and derive our analytical results. Section 5 gauge the strength of the link between the real rate and the rate of depreciation and Section 6 discusses the proposed mechanism's empirical relevance from a broader historical perspective. Section 7 concludes and provides suggestions for further empirical research.

2 Aggregate movements in the depreciation rate

2.1 Measuring capital depreciation

From a national accounts perspective "depreciation" is defined as the change in the value of a capital good associated with the aging of the asset (Fraumeni, 1997). When a capital asset ages its value may change for several reasons. For one, physical wear and tear which cause the productive capacity of a capital good to decline make it less valuable as it ages. In addition, the value of an asset can change due to inflation, revaluation, or other factors that may be correlated with the age of the asset.

In practice, the rate of capital depreciation can be estimated for an individual type of capital good using the regression-based approach pioneered by Hulten and Wykoff (1981). By employing data on the resale price of assets the effect of aging can be separated from pure time effects caused by inflation, and if data is available, "vintage" effect can be controlled for as well.[7]

The Bureau of Economic Analysis (BEA), for example, distinguishes between more than 250 different asset types. While the depreciation pattern for a particular vintage of a capital good is assumed to be constant over time the depreciation profile may differ across different vintages of capital goods (BEA, 2003, p. 29).

discussion below.

[7]The precise shape of the link between the age of an asset and its price can in addition be used to assess which type of depreciation seems to be occurring. For example, if capital depreciation is geometric one would expect the resale value of the asset to decline geometrically with the age of the asset; geometric depreciation is often difficult to reject (Hulten and Wykoff, 1980). Missiles and nuclear fuel rods are exceptions and follow a straight line pattern; see BEA (2003).

Ultimately, BEA and other statistical agencies use the estimates for depreciation to construct net capital stocks and thus consumption of fixed capital in national accounts (Katz and Herman, 1997). Combining depreciation rates with investment data, the perpetual inventory method is employed to construct net capital stocks. The net stock of capital in a particular year is the difference between accumulated past gross investment and the value of accumulated depreciation, and the calculation is conducted at the type-of-asset level of detail.

Finally, to calibrate the average depreciation rate many macro applications have traditionally used the weighted average of the underlying (estimated) depreciation rates where the individual weights are the *real shares* of the individual capital stocks. This can be accomplished by "inverting" the aggregate capital accumulation equation where investments and capital are in constant prices (e.g., Cooley and Precott, 1995):

$$\delta_t = \frac{I_t - \Delta K_t}{K_{t-1}}.$$

The procedure has a couple of drawbacks, however. First, using real shares as weights the time path of the average depreciation rate becomes sensitive to the choice of the base year, just as the average growth of fixed-price GDP is sensitive to the choice of the base year. Specifically, if high depreciation assets have exhibited declining prices over time (e.g., computers), and the final year is used as a base year, the real share in the initial year is lowered which mechanically tends to produce a positive trend (Oulton and Srinivasan, 2003). Second, if investments are chain-weighted it is no longer true that the above approach produces a weighted average of the underlying depreciation rates (Whelan, 2002). Once again the "traditional" approach might lead to an artificially upward trending average depreciation rate, Whelan (2002). To avoid these drawbacks Whelan (2002) and Oulton and Srinivasan (2003) recommend using *nominal shares* as weights when one calculates the average depreciation rate.

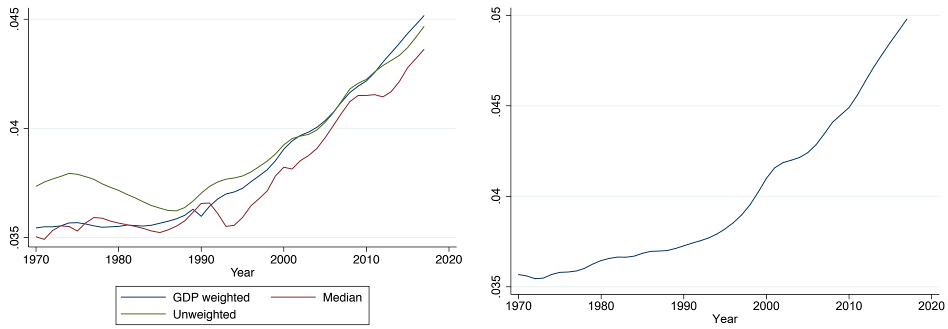
2.2 The evolution of average depreciation: A cross country perspective

We start by exploring the evolution of average depreciation for the bloc of Advanced Economies (AEs). The data on average depreciation rates derives from Penn World Tables (PWT, Feenstra et al., 2015).[8] In the cross-country context, we focus on the GDP-weighted average depreciation rate for the group of Advanced Economies (AE), as defined by the IMF, from 1970 until today. Aside from the weighted average we also calculate the simple average and the median.[9] We focus on AEs because Rachel and Summers (2019) recently estimated the natural rate of interest for this group, as noted in the Introduction, documenting a decline since 1970.

The result is depicted in Figure 1.A. While the average depreciation rate is relatively flat from 1970 to 1990, it has increased by about one percentage point since 1990. As can be seen, the simple average and the median move in a similar way suggesting this pattern is pervasive for the group of countries in

[8]Ideally one would like to use detailed national accounts data for each country. But since the practices regarding depreciation are not fully aligned across statistical agencies (some may use linear depreciation for parts of the period in focus, for example) we resort to Penn World Tables (PWT, Feenstra et al., 2015) where the average depreciation rate is computed at a consistent basis. The cost is a smaller selection of individual assets; PWT distinguishes between nine different asset types. Depreciation is assumed to be geometric and constant across countries and time (cf Inklaar et al, 2019, Table 3).

[9]The PWT does not have information on GDP for all AEs throughout all years since 1970. We, therefore, exclude the Czech Republic, Estonia, Latvia, Lithuania, San Marino, the Slovak Republic, and Slovenia from our sample. These countries constitute less than 2 percent of GDP in 2017.



(a) Advanced Economies

(b) United States

Figure 1: The Depreciation Rate for Advanced Economics (Source: PWT 9.1)

focus. With an eye to the analysis in the next section, where we focus on US experiences, panel B of the figure depicts the evolution of average depreciation in the US according to PWT. The path is similar to that detected for the AE group as a whole, albeit the recent increase is greater than that for the AE group.

Why has the depreciation rate increased? In the case of the PWT, where the capital stock comprises nine types of capital with depreciation rates assumed constant and identical across countries, the reason is a given: the composition of the capital stock has changed. Over time an increasing fraction of the capital stock consists of short-lived assets, such as ICT and software. At the same time, it is important to note that PWT calculates the average depreciation rate using real shares as weights (e.g. Inklaar et al., 2019). This has the unwelcome implication that the trend observed in Figure 1 is sensitive to the choice of the base year.[10]

To better understand the movements in average depreciation we, therefore, turn to data for the US. This change in perspective will allow us to explore the evolution of average depreciation using nominal shares as weights (Whelan, 2002; Oulton and Srinivasan, 2003). It will also allow us to perform a detailed decomposition analysis designed to shed light on the underlying drivers of the long-run evolution of average depreciation.

2.3 The evolution of average depreciation: US data

Based on BEA data, Figure 2 shows the evolution of the average depreciation rate for the United States from 1970-2020 (weighted by nominal capital). As is clear the overall increase in average depreciation for the US is a bit more modest than suggested by PWT data, and the time path is somewhat different. Using the BEA data, depreciation rises from the early 1970s until the early 1990s and stays more or less flat until 2020, compared with a more gradual increase for the PWT data. Quantitatively the average depreciation rate rises by about one percentage point from 1970 to 2020.

Using the richer BEA dataset it is possible to analyze the proximate sources of this increase at a higher level of resolution. Specifically, to explore the source of rising depreciation we use the detailed subdivision of 72 asset types. We next calculate the overall depreciation rate as $\delta_t = \sum_i \delta_{i,t} K_{i,t} / K_t$, where

[10]Though the PWT relies on 9 underlying asset categories, the publicly available data only has four: Structures, machinery, transport equipment, and others. Since the detailed BEA data demonstrates that software and IT are particularly important we do not undertake an exercise where we weigh PWT data by nominal assets.

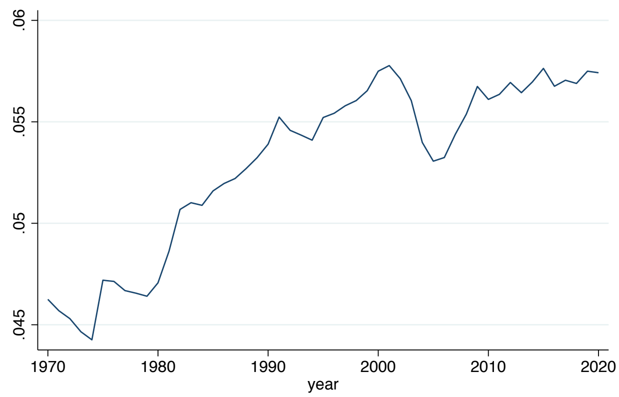


Figure 2: The Depreciation Rate for the United States (Weighted by Nominal Capital. Source: Bureau of Economic Analysis)

$K_{i,t}$ is the nominal (current-cost) stock of private fixed asset of type i (K_t is the total stock) and $\delta_{i,t}$ is the depreciation rate (deprecation divided by stock of capital for each asset type). Finally, we perform a decomposition of the change in the deprecation rate between period $t-s$ and t as:[11]

$$\delta_t - \delta_{t-s} = \underbrace{\sum_i \delta_{i,t-s} \left(\frac{K_{i,t}}{K_t} - \frac{K_{i,t-s}}{K_{t-s}} \right)}_{Reallocation} + \underbrace{\sum_i \frac{K_{i,t}}{K_t} \left(\delta_{i,t} - \delta_{i,t-s} \right)}_{Change\ within\ asset\ type},$$

where the first term gives changes in the aggregate depreciation rate from differential growth in asset classes with different depreciation rates, and the second term considers changes in the depreciation rate within asset types.

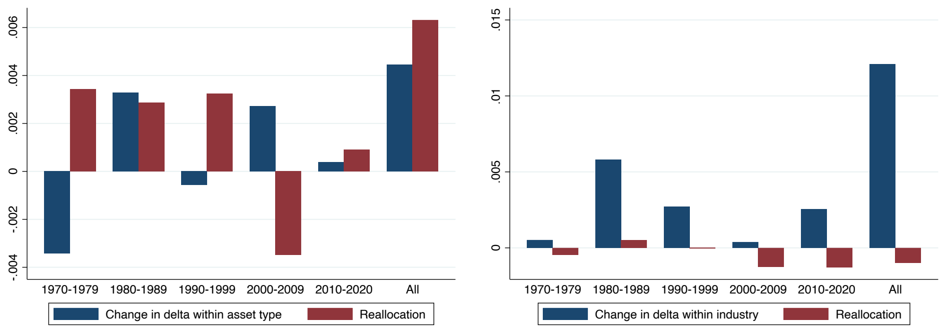
Figure 3.A performs this decomposition for each one of the decades from 1970 to 2020 as well as the whole period. Though there has been substantial variation over the decades, the two terms are of roughly equal importance.[12] Since the 72 asset types used for the calculations here are themselves aggregates of finer asset types, this estimate presents a lower bound on what the reallocation effect would be from a finer disaggregation. ICT and software are important drivers of the increase in depreciation: Whereas the increase in depreciation across all assets is 1 percentage point, it is 0.8 when excluding ICT and 0.2 when excluding both ICT and software.

As can be seen from the figure the analysis suggests a roughly 50/50 split between reallocation and change within asset types. This raises the question of whether the observed movements are related to reallocation across industries, or if the movements in average depreciation are a within-sector phenomenon.

Panel B of Figure 3 shows an analogous decomposition across 18 industries. The figure shows that the entire change comes from within-industry changes with a minor negative contribution from reallocation. As a result, the underlying changes in the capital stock which account for the rise in depreciation is a pervasive phenomenon across sectors.

[11] Autor, Dorn, Katz, Patterson and Van Reenen (2020) use the same approach to decompose the change in the labor share amongst firms.

[12] The negative reallocation effect from 2000-2009 is primarily due to a peak in ICT/software capital around the Dot-com boom around the year 2000. There is some contribution from growing real estate capital until 2009 as well.



(a) Decomposition across asset types

(b) Decomposition across industries

Note: Accumulated changes for decades do not add up to change over the whole period

Figure 3: Decomposition of Change in Depreciation Rate - by decades and whole period

2.4 Discussion

The average rate of capital depreciation has risen since 1970 in the US. In light of the similarity in the trajectory for average depreciation for the US and the bloc of AEs, according to PWT, the increase is probably a pervasive phenomenon. The increase is particularly marked in the beginning of the period flattening out in the end.[13]

As discussed above, average depreciation rates are affected by both physical "wear and tear" and revaluation; if the (relative) price of used capital declines it shows up as faster capital depreciation. When the resale value of capital falls it may however also reflect that investment goods are firm- or sector-specific. Studying aerospace plant closings Ramey and Shapiro (2001) find evidence that investment specificity appears to be important in practice. But since inter-sectoral reallocation seems to be of minor importance to the rise in average depreciation this latter channel is probably not paramount to the issue in hand. This leaves accelerated physical decay and revaluation.[14]

There is no general way in which to fully separate "wear and tear" from revaluation, (Hall, 1968), so the data may be taken to imply that over time the capital stock increasingly consists of capital types featuring faster physical depreciation, revaluation or both. In the context of IT equipment, the revaluation channel is most likely relatively more important (Geske et al, 2007), albeit age-related effects also seem to matter (Doms et al., 2004).[15] What does economic theory predict should be the consequence on the

[13] The time path of depreciation is markedly different from what emerges using PWT due to the change in weighing; real weights in PWT, and nominal weights here. A big part of the increase, as discussed above, is due to ICT and software which usually is viewed as a general purpose technology. An ad hoc way to mimic the arrival of a general purpose technology within the standard Solow model is to simultaneously vary total factor productivity and the depreciation rate (see Aghion and Howitt, 1998, Ch. 8.4). The first parameter change is meant to capture the long-run increase in productivity, and the second captures the initial slump, originating from obsolescence of machines embodying the old technology. The observed time path of depreciation for the US is thus broadly consistent with the first part of this kind of rough narrative, bearing the great productivity slowdown in mind which emerges in the early 1970s.

[14] The study by Ramey and Shapiro (2001) also contributes to the literature on capital depreciation in that they have access to the actual purchase price of the equipment, which otherwise is assumed in the literature to be identical to the list price. Moreover, by studying a natural experiment (plant closings) their sales data should be free of the "lemons problem" which refers to the concern that part of the price reduction that the literature classifies as depreciation may be due to selection. Reassuringly their estimates are fairly similar to existing estimates in the literature.

[15] As pointed out by the authors, this channel undoubtedly captures more than physical decay. E.g., if new software becomes progressively harder to run on existing IT equipment this will show up as an "age effect", which illustrates the point that economic and physical depreciation is hard to disentangle in practice.

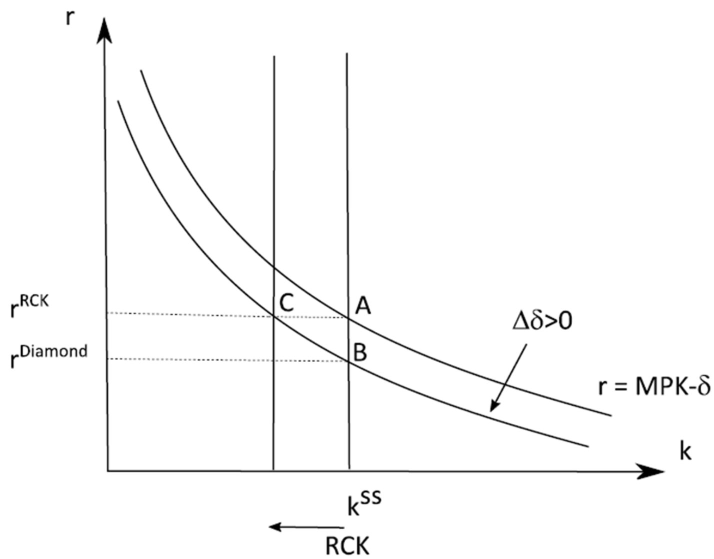


Figure 4: Equilibrium in the Capital Market

steady-state real rate of interest?

3 Depreciation and the real rate in standard growth models: A review

The analysis in this paper will be conducted within a closed economy setting since we are pondering the causes of what is likely an international phenomenon occurring across all AEs. When considering economic areas of this size as a bloc the closed economy assumption is a common analytical simplification. More importantly, Rachel and Summers (2019) recently documented that for the group of AEs the consolidated current account has been more or less in balance since the end of the 1970s. As a result, total investments in the area have de facto been constrained by total savings, which justifies the closed economy assumption.

Figure 4 illustrates the instantaneous equilibrium in the capital market of a neoclassical growth model for a competitive one-sector economy. It involves two schedules: the downward-sloping demand for capital from profit maximization which equates the real rate of return to the net marginal product of capital, and the instantaneous supply of capital which is fixed in the short run. To begin we will focus on the impact of physical depreciation and then turn to revaluation.

If capital depreciation rises, the demand for capital shifts inwards due to a declining net marginal product. This produces a force that pushes down the real rate of interest, as illustrated in Figure 5. But the long-run (steady-state) impact depends on how the supply of capital subsequently changes.

Suppose we initially are located in the steady state of a Ramsey-Cass-Koopmans (RCK) model. An increase in the rate of capital depreciation will reduce the long-run stock of capital in efficiency units ensuring that in the new steady state the real rate of return is left unaffected; that is, the economy moves from point A to point C in the figure. Another way to state this result would be to say that the supply of capital is perfectly elastic in the long run, which would amount to assuming a horizontal supply curve - in this case going through both points C and A.

Suppose, alternatively, that we are in a steady state of a Diamond model. If the savings rate of the young is insensitive to the real rate of return (because preferences are logarithmic, say) the steady-state capital stock is unaffected by an increase in the depreciation rate. As a result, in the new steady state the real rate is lowered by the increase in the depreciation rate on a 1:1 basis; the economy moves from point A to point B in the figure.[16]

Another way to state the latter result would be to say that the supply curve is vertical - in this case going through both points A and B. Of course, log preferences can be viewed as an extreme assumption. If the savings rate of the young is rising in the real rate of return, the supply curve is upward-sloping rather than vertical. Higher depreciation still lowers the steady-state real rate, but less so. The preceding remarks are demonstrated more formally in the appendix.[17]

Why do the conclusions differ so markedly? The reason is that the two models fundamentally differ in terms of how savings are related to the factor income distribution; in a RCK model, savings depend on capital income whereas savings depend on wage income (which accrues to the young) in the Diamond setting (Bertola, 1994). A higher depreciation rate serves to lower capital income which ultimately lowers the long-run supply of capital and causes the RCK economy to move from point B to C in transition from the old steady state to the new. This does not occur in a Diamond model where savings depend on wage income.

At a deeper level, the difference in the response of capital accumulation to changes in physical depreciation is caused by differences in the savings *motive*. In a pure Diamond model, the life cycle motive is all-important which therefore leaves savings chiefly dependent on (lifetime) wage income. When the OLG framework is extended by altruistic preferences Barro (1974) shows that the resulting model is dynamically isomorphic to an RCK model. Hence, the key reason why capital income matters is the bequest motive's importance to savings.

This means that in forming expectations about how the long-run supply of capital changes when capital depreciation rises, the relative importance of bequest in total savings is the key question. In the next section, we develop a "hybrid-model" where both the life-cycle motive and the bequest motive are present and demonstrate this more formally.[18]

Though this discussion focused on physical capital, as discussed above the observed rise in depreciation is equally likely to be the result of revaluation considering the key role played by IT, software etc. in the observed increase in average depreciation. Karabarbounis and Neiman (2014) show an accelerating decline in the global price of investment goods from 1980 since when real prices have declined by around 2 percent a year to 2010. This would depreciate the value of existing capital goods.

To see the influence from revaluation in a simple way, suppose we extend the above analysis to a two-sector setting distinguishing between investment and consumption goods. We normalize the price of consumption goods to 1, and to simplify matters, consider a one-period problem facing a profit-

[16] A Solow-Swan model contains an interesting third type of prediction. If the economy is precisely in the golden rule steady state changes in capital depreciation leaves the steady state real rate unaffected, but if the economy is dynamically efficient it increases the steady state real rate, and vice-versa if it is dynamically inefficient. See the appendix for details.

[17] Moll, Restrepo and, Rachel (2021) develop a model with an upward-sloping supply curve of capital in the interest rate arising from a specific dissipation shock to wealth accumulation and argue that a broad class of models would feature upward-sloping supply curves. Though our focus here is on the savings motive, other models with an upward sloping supply curve would also find that a higher depreciation rate lowers the real interest rate.

[18] It would appear to be uncontroversial to assert that *both* motives matter in practice (Dynan et al., 2002), albeit the relative strength of the two motives remains a matter of debate (De Nardi et al. , 2016).

maximizing representative firm. Profits are given by

$$\Pi = F(K_t, A_t L_t) + p_t (1-\delta) K_t - w_t L_t - (1+r_t) p_{t-1} K_t,$$

where F is a CRS production function, p_t is the price of investment goods in period t, K_t and L_t are factor uses, w_t is the wage and r_t the real interest rate. A_t is a technology parameter. At the beginning of the period, the firm can be thought to issue corporate bonds, which are purchased by investors. This enables the firm to acquire capital for production purposes ($p_{t-1}K_t$). It then produces and pays wages to its employees. At the end of the period, it sells the remaining capital stock (i.e., the fraction $1-\delta$) at the prevailing price (p_t) and retires the corporate bonds. The first-order condition for capital equates the marginal product of capital to the user-cost of capital

$$F_K = p_{t-1}\left[1 + r_t - \frac{p_t}{p_{t-1}}(1-\delta)\right].$$

In the one sector setting this condition reduces to the one featured in Figure 5: $F_K = MPK = r_t + \delta$. But in the two-sector setting, the (relative) price of capital reflects buying and selling of capital may involve either capital gains or losses. In the case where the price of capital is declining over time, relative to the price of consumption, the transaction involves a capital loss which raises the user-cost of capital just like higher physical capital depreciation does. From a partial perspective, therefore, there is no difference between an increase in δ or a fall in p_t/p_{t-1}; both will serve to reduce the demand for capital by forward-looking firms. As discussed in the last section, the rise in NIPA average depreciation is most likely caused by either rising physical depreciation (δ), revaluation ($p_t/p_{t-1} < 1$), or both.

From a broader perspective, however, there is a difference in that the fall in the price of capital is endogenous. It might, for instance, be caused by a faster rate of technological changes in the capital goods-producing sector than what prevails in the consumption goods sector. A faster decline in the (relative) price of capital, therefore, reflects a faster rate of technological change which will matter to capital accumulation in its own right via the long-run growth rate of the economy. But a similar outcome to the one described in the one sector setting can be obtained if this last effect is eliminated.

Below we construct a two-sector model where investment goods and consumption goods are produced with similar technologies, differing only in the rate of technological progress.[19] The steady-state growth rate of the economy combines these two. An increase in the growth rate of technology for investment goods will increase the overall growth rate of the economy, which has not generally been observed. However, a shift in technological growth from consumption goods to investment goods leaves the growth rate of the overall economy constant but drives the prices of investment goods down thus producing the effect on the user-cost of capital discussed above.[20] In this case, the remarks above will carry over: faster economic depreciation will lower the real rate of return in the Diamond setting, but not in the RCK setting. Again, this difference in outcomes ultimately depends on the relative importance

[19] Alternatively, we could have built a vintage capital model along the lines of Phelps (1962). This will give rise to the same result.

[20] This is related, but distinct, from a recent literature arguing that technological change has shifted in favor of automation without increasing the overall growth rate of the economy (Acemoglu and Restrepo, 2019) and Hémous and Olsen (2022). *Automation*, however, is the replacement of labor tasks with tasks performed by capital, whereas faster technological growth in investment goods is the more rapid replacement of capital by new vintages of capital.

of bequest in total savings since this governs how sensitive savings are to change in capital income and therefore depreciation. We develop this intuition further in the next section.

4 The Model

We consider an overlapping-generations economy. Time is discrete $t = 1, 2...\infty$. The economy is closed, and all markets are competitive. The economy comprises two sectors that produce consumption goods and investment goods, respectively. The price of consumption is normalized to one. Both sectors experience exogenous technological change. There is no population growth and the two generations are of equal size and normalized to one. The details are given below.

4.1 Consumers

Individuals live for two periods. In the first period of life, they supply a unit of labor inelastically for which they receive a wage. They receive bequests and save. During old-age individuals consume and pass on bequests to their heir from their savings during youth. Lifetime utility for a generation born at time t is

$$u_t = \ln c_{1,t} + \frac{1}{1+\rho}(\ln c_{2t+1} + \eta \ln b_{t+1}).$$

where $c_{1,t}$ is consumption during youth and $c_{2,t}$ is consumption during retirement, b_{t+1} is bequests while ρ represents time preferences. Per period utility is logarithmic, which implies that the savings rate will be independent of the real rate; we discuss below how our results are likely affected if this assumption is relaxed.

The bequest motive is captured by joy-of-giving preferences, and η parameterizes the strength of the bequest motive. An alternative modeling approach is to assume that households behave dynastically, deriving utility from the utility of the descendants and so on. But this will produce a model where the aggregate savings behavior is isomorphic to an infinitely lived consumer if bequests are passed on, which implies that steady state savings solely depend on capital income (Barro, 1974; Bertola, 1994). In order to produce behavior where both wages and capital income matter, we, therefore, resort to the joy-of-giving specification.[21]

The budget constraints are

$$c_{1t} + s_t = w_t + b_t \equiv I_t,$$

$$(1 + r_{t+1})\, s_t = c_{2t+1} + b_{t+1},$$

where the assumption that each generation is of equal size eliminates any need to normalize first period bequest. Accordingly, the first period income comprises wage income, w, and bequest. Second period income consists of savings during youth with interest, where r_{t+1} is the real rate of interest.

[21] The assumption that consumer welfare depends on terminal wealth, or bequest, goes back to Yaari (1964). The empirical relevance of a bequest motive is well established albeit the strength of the motive, relative to the life-cycle motive, is an active area of research. See De Nardi, French and Jones (2016).

Maximizing life-time utility subject to these constrains leads to the following solutions for first period savings and late-in-life bequest

$$s_t = \frac{1+\eta}{2+\rho+\eta} I_t,$$

$$b_{t+1} = \frac{\eta}{1+\eta} \left(1 + r_{t+1}\right) s_t.$$

Log preferences imply that the savings rate is independent of the real rate of return. The presence of a bequest motive ($\eta > 0$) increases the savings rate compared to a standard two-sector OLG model. Moreover, consumers divide their accumulated lifetime wealth between own old age consumption and bequest for their offspring; the split becomes more favorable to the next generation if η increases in size.

Taken together this behavior implies, in contrast to a Diamond (1965) model with logarithmic preferences, that the real rate of return will influence the process of capital accumulation: A higher real rate increases accumulated savings which translates into greater bequests and thus the income of the next generation which then fuels more savings. The strength of this channel depends on the size of η . As a result, both wage income and capital income will influence capital accumulation in the present setting.

4.2 Production

The economy comprises two competitive sectors that are equipped with Cobb-Douglas production technologies:

$$C_t = K_{c,t}^{\alpha} (A_{c,t} L_{c,t})^{1-\alpha}, \tag{1}$$

$$I_t = K_{I,t}^{\alpha} (A_{I,t} L_{I,t})^{1-\alpha}. \tag{2}$$

C is aggregate consumption, I is aggregate investment, K is capital, L is labor and A is technology. We assume technology progresses at an exogenous rate, and that the speed of technological change is faster in the investment good sector

$$\frac{A_{c,t+1}}{A_{c,t}} = 1 + g_c < \frac{A_{I,t+1}}{A_{I,t}} = 1 + g_I.$$

The Cobb-Douglas assumption plays an important role in eliminating the standard reason why a decline in the relative price of investment should matter to the real rate of interest. The standard mechanism works as follows. When the price of investment declines it affects investment demand in two ways, for given amounts of savings. Holding the marginal product of capital constant a reduction in the price of investments will increase the return to investments 1:1 as a given amount of savings buys more capital. But, holding savings constant, more investment works to lower the marginal product of capital and hence the return to investment. The net effect on investment demand will be negative (positive) if the elasticity of substitution between labor and capital is below (above) one (Sajedi and Twaites, 2016; Rachel and Smith, 2017). Hence, the Cobb-Douglas assumption, featuring an elasticity of substitution of one, ensures that the standard mechanism does not influence the real rate.

The period t decision problem of the representative firm in the consumption goods sector is

$$\max K_{c,t}^{\alpha}(A_{c,t}L_{c,t})^{1-\alpha} + (1-\delta)\, p_t K_{c,t} - K_{c,t}p_{t-1}\,(1+r_t) - w_t L_{c,t},$$

where p is the price of investment goods in terms of consumption goods. To fully understand this profit function a few remarks are warranted.

In any given period, say period $t-1$, a young generation is saving. At the end of the period, these individuals lend their savings to upcoming firms that wish to produce during period t, either consumption goods or investment goods. With these funds, the firm acquires capital at the going rate, p_{t-1}. When period t dawns the producers use their acquired capital along with labor (hired at the wage w_t) to produce their output. They then sell the non-depreciated capital (the fraction $1-\delta$) at the going price, p_t, and pay back savers the sum borrowed with interest. This timing implies that changes in the relative price of investment across periods will influence the user-cost of capital: if the price goes up the firm obtains a capital gain, and if the price declines - as is the case in the present model - the transaction involves a capital loss.

The first order conditions are

$$p_{t-1}\left[(1+r_t) - \frac{p_t}{p_{t-1}}(1-\delta)\right] \equiv u_t = \alpha K_{c,t}^{\alpha-1}(A_{c,t}L_{c,t})^{1-\alpha},$$

$$w_t = (1-\alpha)\, K_{c,t}^{\alpha}(A_{c,t})^{1-\alpha}(L_{c,t})^{-\alpha}$$

where u_t is the user-cost of capital which comprises the purchase price of capital, p_{t-1}, the cost of borrowing, r_t, the cost of physical depreciation, δ, and the capital losses due to a declining price of investment, $p_t < p_{t-1}$. The corresponding conditions for the investment goods producer are

$$u_t = p_t \alpha K_{I,t}^{\alpha-1}(A_{I,t}L_{I,t})^{1-\alpha}, w_t = p_t\,(1-\alpha)\, K_{I,t}^{\alpha}(A_{I,t})^{1-\alpha}(L_{I,t})^{-\alpha}$$

where the unit cost of the inputs equal the value of the marginal product of capital and labor, respectively. Since the factor prices faced by produces are the same factor intensities are equalized:

$$\frac{w_t}{u_t} = \frac{1-\alpha}{\alpha}k_{I,t} = \frac{1-\alpha}{\alpha}k_{c,t}, \tag{3}$$

where $k_{c,t} = K_{c,t}/L_{c,t}$ and $k_{I,t} = K_{I,t}/L_{I,t}$.

4.3 Aggregation

The free mobility assumption and the requirement that both sectors are active implies that the relative price of investment goods in equilibrium is given by

$$p_t = \left(\frac{A_{c,t}}{A_{I,t}}\right)^{1-\alpha}, \tag{4}$$

which along with (3) implies that total output can be written

$$Y_t = C_t + p_t I_t = K_{c,t}^{\alpha}(A_{c,t}L_{c,t})^{1-\alpha} + pK_{I,t}^{\alpha}(A_{I,t}L_{I,t})^{1-\alpha} = K_t^{\alpha}\,(A_{c,t}L)^{1-\alpha}.$$

Moreover, due to competitive markets and constant returns to scale total output equals total cost

$$Y_t = w_t L + u_t K_t$$

where we have used that the user-cost of capital equals the marginal product of capital:

$$u_t = p_{t-1}\left[(1+r_t) - \frac{p_t}{p_{t-1}}(1-\delta)\right] = \alpha\frac{Y_t}{K_t}.$$

As a result the instantaneous equilibrium real rate of return is given by

$$1 + r_t = \alpha\frac{Y_t}{p_{t-1}K_t} + \frac{p_t}{p_{t-1}}(1-\delta), \tag{5}$$

where

$$\frac{p_t}{p_{t-1}} = \left(\frac{1+g_c}{1+g_I}\right)^{1-\alpha} < 1. \tag{6}$$

Hence, keeping $\frac{Y}{p_{t-1}K}$ constant faster technological change in the investment good sector relative to the consumption goods sector, or greater physical decay, lowers the real rate of return. Naturally, in the long run the (nominal) capital-output ratio adjusts as discussed in the next section.

4.4 Macro dynamics

The amount of capital available for future production is given by the (investment value of the) savings of the young

$$p_t K_{t+1} = s_t,$$

where it is recalled that the size of generations has been normalized to one. Substituting for optimal savings as well as (lagged) bequest leads to

$$p_t K_{t+1} = \frac{1+\eta}{2+\rho+\eta}\left[\frac{\eta}{1+\eta}(1+r_t)\,p_t K_t + (1-\alpha)\,Y_t\right].$$

The terms in the square bracket reflects that the income of the young is partly based on capital income, via bequests, and partly labor income. The two motives for savings, life-cycle and bequest, both influence capital accumulation, and their relative importance is determined by η; if $\eta = 0$ only wage income matters to the process of capital accumulation.

If we define the nominal capital stock in efficiency units as

$$q_t \equiv \frac{p_{t-1}K_t}{A_{I,t}^{\alpha}A_{c,t}^{1-\alpha}}$$

it can be shown that the (nominal) capital stock in efficiency units evolves in accordance with the following law of motion:

$$q_{t+1} = \frac{1}{1+g}\frac{1+\eta}{2+\rho+\eta}\left\{\left[\frac{\eta}{1+\eta}\alpha + (1-\alpha)\right]y_t + \frac{\eta}{1+\eta}q_t(1-\delta)\frac{p_t}{p_{t-1}}\right\} \equiv \psi(q_t),$$

where $1+g \equiv (1+g_I)^{\alpha}(1+g_c)^{1-\alpha}$ is the long-run growth rate of output and $\frac{p_t}{p_{t-1}}$ is constant (cf equation 6). Output in efficiency units is

$$y_t = \frac{Y_t/L}{A_{I,t}^{\alpha}A_{c,t}^{1-\alpha}} = q_t^{\alpha}\left(\frac{p_{t-1}}{p_t}\right)^{\alpha}.$$

Hence the law of motion for capital says, via the first term in the curly brackets, that when output increases it enables more capital accumulation since it stimulates both labor and capital income.[22] Greater physical decay or faster investment price deflation reduces savings and accumulation via the second term in the curly brackets as this reduces the net income of the young consumers via less bequest.

The dynamical system admits a unique and globally stable steady state where $q_{t+1} = q_t = q^*$ since $\psi(0) = 0, \psi'(q_t) > 0, \psi''(q_t) < 0, \lim_{q\to\infty}\psi'(q_t) = \infty$ and $\lim_{q\to\infty}\psi'(q_t) = \frac{1}{1+g}\frac{\eta}{2+\rho+\eta}(1-\delta)\frac{p_t}{p_{t-1}} < 1$. It is straightforward to show that in the steady state the ratio of output to (nominal) capital is

$$\frac{y^*}{q^*} = \frac{(1+g)(2+\rho+\eta)}{\eta\alpha+(1+\eta)(1-\alpha)} - \frac{\eta(1-\delta)\frac{p_t}{p_{t-1}}}{\eta\alpha+(1+\eta)(1-\alpha)}, \tag{7}$$

In the steady state a declining price of investment goods serves to increase the average productivity of capital due to accelerating economic depreciation, capturing physical decay as well as revaluation. This only happens when the bequest motive is operative, as can be seen. The underlying cause is that when economic depreciation increases it reduces the return to savings, thus bequests and therefore the income of the young consumers. Capital accumulation is thereby reduced and this increases long-run average capital productivity.

The reason why relative prices appear nowhere else in the formula *except* via depreciation is due to the Cobb-Douglas production technology which features an elasticity of substitution between capital and labor of one. The two "traditional effects", discussed above, whereby a given amount of savings buys more capital – thus stimulating investments - and a declining marginal product of capital - thus discouraging investments - exactly cancel out.

4.5 Analytical results

Using equations (5) and (7) the steady state real rate of return is

$$1+r^* = \alpha\frac{y^*}{q^*} + \frac{p_t}{p_{t-1}}(1-\delta) = \frac{\alpha(1+g)(2+\rho+\eta)}{\eta\alpha+(1+\eta)(1-\alpha)} + \frac{(1+\eta)(1-\alpha)}{\eta\alpha+(1+\eta)(1-\alpha)}\frac{p_t}{p_{t-1}}(1-\delta), \tag{8}$$

which is the first key result of the analysis.

When economic depreciation accelerates, either because δ rises or $\frac{p_t}{p_{t-1}}$ falls, the real rate declines. When depreciation rises it lowers the return to investment directly, but it also stifles capital accumulation if the bequest motive is operative ($\eta > 0$) which serves to increase the long-run return via a rising average product of capital in the steady state. The latter effect serves to partially counteract the direct impact of

[22] Output rises when the price of investment (relative to consumption) declines. The latter appears for the following reason. The relative price of investment that matters to output in period t in a NIPA sense is p_t,via the expenditure identity, but the amount of capital in use during period t was bought at the price p_{t-1}. When the price of investment decline, for nominal capital to output fixed, the amount of output produced in period t units of consumption goes up. In consumption terms there is a "capital gain" from the reduction in investment prices.

depreciation. In the special case where bequest motive is absent ($\eta = 0$) the pass through of depreciation on the real rate is 1:1. Naturally, if the decline in p_t/p_{t-1} follows from an increase in $1 + g_I$ (equation 6) the growth rate of the economy, g, would grow as well. This is not consistent with general trends in TFP growth. A natural exercise is therefore to increase in g_I and lower g_c so as to keep the overall growth rate constant.[23]

The result relies on the "double Cobb-Douglas" assumption: preferences, as well as production technology, is Cobb-Douglas. The Cobb-Douglas assumption in production is important to highlight that the standard reason why declining investment prices may lower the real rate is not present, as discussed above. The Cobb-Douglas assumption in preferences is more controversial. But it serves to create a tight link between the bequest motive and the sensitivity of savings to the real rate: the only reason why the real rate matters to savings (in the model) is due to the bequest motive.

More generally the savings *rate* may well respond to changes in the real rate. The direction of change is determined by the elasticity of intertemporal of substitution (EIS): if EIS is larger than one a greater real rate means more savings (substitution effect dominates) and vice-versa if EIS is smaller than one (income effect dominates). While the size of EIS is an active area of research recent estimates lean towards an EIS smaller than one (e.g. Best et al., 2020). In the present context, this means that deviating from log utility should (if anything) *increase* the negative impact of depreciation on the real rate. In the model above, faster depreciation lowers capital accumulation via fewer bequests. But this countervailing effect on the steady state real rate will be mollified by an *increase* in the savings *rate* if EIS is smaller than one. Accordingly, the assumption of log utility leads to results that - if anything - underestimate the effect of depreciation on the steady state real rate.[24]

The second key result is that the detected mechanism serves to increase the gap between the marginal product of capital and the real rate. This is apparent from equation (8) which in the steady state can be restated as

$$1 + r^* - \alpha \left(\frac{y}{q}\right)^* = \frac{p_t}{p_{t-1}} (1 - \delta)$$

When either physical decays increases, or the price of investments decline the marginal product of capital rises relative to the real rate of return. Or, equivalently, while the marginal product of capital rises the real rate declines.

5 Gauging empirical magnitudes

To assess the impact of depreciation on r^* we need to pin down a reasonable value for η. In this respect, it is useful to note that the model predicts that the fraction of the total capitalized value of the capital stock that is made up of bequests is constant at all points in time:

$$\frac{B_t}{(1 + r_t)K_t} = \frac{\eta}{1 + \eta}. \tag{9}$$

[23] If anything TFP growth has exhibited a tendency to decline since 1970 (e.g., Jones, 2016). This can be accomplished in the model by unilaterally decreasing g_c which simultanously works to reduce $\frac{p_t}{p_{t-1}}$. In the end the real rate declines via rising depreciation *and* slower overall growth.

[24] From a more technical perspective, however, the dynamic analysis of the two sector model becomes much more complicated if savings are declining in the real rate potentially leading to indeterminacy (cf. Galor, 1992)

Recently, Alvaredo, Garbinti and Piketty (2017) estimate, drawing on what is essentially an *accounting* analysis developed in Piketty (2011), that $B_t / [(1+r_t)K_t]$ falls in the $0.5 - 0.6$ range for the US and Europe today. Hence, a value of η slightly above 1 is reasonable.[25]

As noted above the relative importance of the bequest motive and the life cycle motive in savings remains an active area of research (De Nardi et al., 2016). From this perspective the present calibration is crude. In practice, some observed bequests may well be unintentional for which reason we *overestimate* the role played by the bequest *motive* by proceeding the way we do in the remaining. But the fact remains that even if unintentional, bequests will serve to establish a link between capital *income* and savings (of the next generation), which is what matters in the present setting. In this sense, the "mistake" we make in exaggerating the role of the bequest motive is not so important for the issue at hand.[26]

If we therefore take $\eta = 1$ and furthermore assume that the share of capital $\alpha = 0.4$, the "dampening factor" on the depreciation rate in equation (8) is

$$\frac{2(1-0.4)}{0.4+2(1-0.4)} = 0.75.$$

Hence, if δ increases by 1 percentage point, it would reduce the steady-state real rate of return by about 75 bps. Naturally, as discussed in Alvaredo et al. (2017) there is some uncertainty about the estimated magnitude of bequests which means the calibrated value for η is equally uncertain.

To develop a more robust sense of magnitudes, it is useful to note that the model allows us to bound the influence from depreciation. When bequests are unimportant - $\eta = 0$ - we recover the 1:1 association from a pure Diamond setting. At the other extreme, when bequests become the key savings motive, we obtain a dampening factor on the influence from depreciation. Specifically

$$\lim_{\eta \to \infty} \frac{(1+\eta)(1-\alpha)}{\eta\alpha + (1+\eta)(1-\alpha)} = 1 - \alpha.$$

Hence, given $\alpha = 0.4$, the bequest-augmented Diamond model predicts that if δ rises by one percent, the steady-state real rate declines by 60 bps. Accordingly, the influence from depreciation is bounded in an interval from 0.6 to 1, with 0.75 as a reasonable best guess.

Where does this leave us in accounting for post-1970 developments in the real rate of return? The recent work by Rachel and Summers (2019) estimates for the group of Advanced Economies that the natural rate of interest has declined by about 300 bps since 1970. As discussed earlier, it is difficult to find comparable numbers for the Advanced Economies as a whole, but if developments in the US are a reasonable guide, the depreciation rate has increased by about one percentage point since 1970 (cf. Section 2). This should, according to the model above, thus serve to reduce the real rate by between 60 bps and 100 bps, or what amounts to between $1/4$ to $1/3$ of the secular decline in the natural rate.

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[25] While the methodology does require assumptions (or, if available, estimates) for the extent of *inter vivos* transfers from parents to children, it does not involve theory-based assumptions; it is not a "model specific" calibration. As a result, no theoretical tension arises when we employ the estimates in our setting.

[26] Another issue may be how to think about the precautionary motive which undoubtedly plays a role in practice albeit does not matter in the present model due to the absence of uncertainty. The precautionary motive serves to raise overall savings but should in itself have no implications for the link between *factor income* and savings. In principle one can view the present top down calibration approach as one whereby the role played by real world uncertainty (as it manifests in observed bequests) is evenly divided between the bequest and life-cycle motive.

6 External validity of the role played by depreciation

Recently evidence has emerged that the post-1970 trend in the real rate might be part of a "super-secular" pattern. Schmelzing (2019) shows that carefully assembled data for the evolution of the real rate of interest exhibit a persistent negative trend since the 14th century. During the post-Napoleonic period (1820-), for example, Schmelzing documents that the trend amounts to an annual decline of 2.29 bps per year on average, which accumulates to a 4.6 pp decline in the real rate until the present day.

As observed in the Introduction, various factors have been put forward as candidate explanations for the recent evolution of the real rate of interest, including changes in productivity growth, demographics, and more. Yet, the literature has questioned the external validity of some of the explanations bearing the longer run perspective in mind. For instance, Borio et al. (2017), using panel data from 1870 onward for 19 countries, find that several variables believed to be key drives of changes in the real rate often seem not to co-vary with real rates in an expected way. A similar pattern is uncovered by Lunsford and West (2019) who explore the determination of the real rate in the United States since 1890 using time series methods. Though some standard demographic determinants are correlated with the real rate in the expected way most other variables exhibit a mixed relationship to the real rate.

This raises the question of whether the mechanism under scrutiny suffers an "external validity" issue? To put it differently: Is there a reason to believe that depreciation could have played a role in the long-run pattern uncovered by Schmelzing (2019), in addition to the more recent (post-1990) decline in the real rate?

Answering the latter question rigorously would require data on the composition of the capital stock and estimates of the depreciation rate by asset class for the early 19th century. Perhaps future work by economic historians will produce this kind of data. For now, we will have to resort to more circumstantial evidence.

The point we will make is that since the relative strength of the two savings motives in focus here (life cycle vs. bequest) likely changed over the period the *pass-through* of the depreciation rate likely changed as well. This change should put a downward force from the depreciation rate on the real rate *even if* the depreciation rate itself remained constant over the period (see equation 8). We reiterate that our point is that the strength of the negative force *from depreciation* on the real rate increases when the savings motive changes. Whether the structural shift in the savings motive *overall* has served to lower the real rate is a different proposition that we do not posit; we only focus on the partial influence from the depreciation rate.

DeLong (2003) argues that before the Industrial Revolution bequest played a much more crucial role in the process of capital accumulation than it does today. More concretely, DeLong argues that while the share of bequests in total wealth is perhaps about 0.4 today (a bit lower than the more recent estimate by Alvaredo et al. (2017)) the figure is likely to have been closer to 0.9 in the pre-industrial world. Broadly consistent with these views Alvaredo et al. (2017) find a bequest share of about 0.5 in 21st century Europe and between 0.7 and 0.8 in 1900. In the US, the pattern is more ambiguous and seems to exhibited a wave-like pattern from 1900 to 2010, starting and ending at a bequest share between 0.5 and 0.6.

Suppose we take DeLong's estimate for the pre-industrial setting as a rough proxy for the situation in the early 19th century, and the recent estimates from Alvaredo et al. (2017) as our endpoint estimate for the share of bequests. Considering equation (8) we would then expect the "depreciation channel" to be modified in the following way:

$$\Delta r^* = -\left[\frac{(1+\eta^{2020})(1-\alpha)}{\eta^{2020}\alpha + (1+\eta^{2020})(1-\alpha)} - \frac{(1+\eta^{1820})(1-\alpha)}{\eta^{1820}\alpha + (1+\eta^{1820})(1-\alpha)}\right](1-\delta)\frac{p_t}{p_{t-1}},$$

assuming a constant rate of depreciation throughout the period and share of capital. Now, suppose the share of capital is 0.4, and note that DeLong's calculations suggest a value for $\eta^{1820}$of about 9, while Alvaredo et al. (2017) suggest a value of $\eta^{2020}$of about 1. This means a change in the partial effect of the depreciation rate of $\Delta r^* \approx -0.125\delta$. To obtain an implied decline in the real rate we need a reasonable guess for average depreciation over the period, which we cannot know. But if we disregard the recent run-up in average depreciation, documented above, a value of about 3.6 percent may be reasonable. In this case, theory would suggest a resulting decline in the real rate of roughly 45 basis points, which is about ten percent of the observed decline since 1820, according to the data assembled by Schmelzing (2019).

Naturally, if the average depreciation rate in the 21st century is *higher* than during the 19th century, this number will increase. Across this time horizon, it may well have increased due to changes in the composition of assets. Going sufficiently far back in time, most of the capital stock would probably be structures. Over time equipment would matter progressively more, just as ICT-related capital assets have risen in importance recently. Assessing the quantitative significance of such changes for the real rate - if they occurred - will, however, have to await the availability of the relevant data.

From this discussion, two conclusions emerge. First, the mechanism that we have explored in the present paper appears consistent with long-term trends. Second, while the mechanism may contribute to an understanding of the forces that have been responsible for a "supra-secular" decline in the real rate of interest, it leaves much unexplained.

7 Concluding remarks

Judged from the best available evidence, the average rate of capital depreciation has increased over the last half-century in the US and it seems likely that this trend is pervasive across advanced economies. A decomposition analysis finds that a rising share of capital assets featuring relatively high depreciation, such as IT, is chiefly responsible. This development is pervasive across sectors; the average is not going up because of reallocation across sectors.

From a theoretical perspective a rising depreciation rate, whether driven by rising physical decay or revaluation, will work to lower the real rate of return in the steady state if aggregate savings are only modestly affected by changes in capital income. This amounts to saying that if the life-cycle motive is sufficiently important in explaining aggregate savings a rising depreciation rate should work to lower the real rate in the long run. While the relative importance of the life cycle and bequest motive for savings remains an active area of research a rough top-down calibration suggests that the highlighted mechanism may have contributed significantly to the reduction in the "natural" real rate of return since 1970. Aside from contributing to the secular decline in the real rate, a rising depreciation rate could also contribute to a greater spread between the calibrated marginal product of capital and the risk free real rate of interest.

Capital depreciation reflects both physical decay and revaluation. The revaluation element in turn reflects that declining investment prices represent a capital loss for users of capital, which serves to lower the return *ceteris paribus*. The present study can thus be seen as adding a complementary reason why declining investment prices may lower the real rate of return beyond that which has been explored in the literature. Declining investment prices may thus be considerable more important in explaining developments with respect to the real rate, in totality, than hitherto recognized. If productivity growth in the production of investment goods is expected to be above that in the production of consumption goods in the future, a declining investment price may serve to keep the real rate of return low in the future.

Finally, while the calibration of this paper suggests that an increasing rate of depreciation has had an important impact on the real interest rate, a direct test of the proposed mechanism would be useful. As should be clear, if the return on savings decline the relative consumption opportunities of the old and young are affected, since the latter group also fuels their consumption via wage income. Accordingly, if rising depreciation leads to a lower return on savings, on net, consumption of the young should increase *relative* to that of the old. Whether rising depreciation has worked to lower relative consumption of the old across Advanced Economies over the last half century appears to be an interesting question for empirical research to resolve.

References

[1] Acemoglu, D. & Restrepo, P. (2019). Automation and New Tasks: How Technology Displaces and Reinstates Labor. Journal of Economic Perspectives, 33(2), 3-30

[2] Aghion, Philippe, and Peter Howitt. "Endogenous Growth Theory Cambridge." MA: MIT Press, 694p (1998).

[3] Alvaredo, F., Garbinti, B., & Piketty, T. (2017). On the share of inheritance in aggregate wealth: Europe and the USA, 1900--2010. Economica, 84(334), 239-260

[4] Ashraf, Q., & Galor, O. (2011). Dynamics and stagnation in the Malthusian epoch. American Economic Review, 101(5), 2003-41.

[5] Autor, D., Dorn, D., Katz, L., Patterson, C., Van Reenen, J. (2020) The Fall of the Labor Share and the Rise of Superstar Firms, Quarterly Journal of Economics, 135(2)

[6] Barro, R. J. (1974). Are government bonds net wealth?. Journal of political economy, 82(6), 1095-1117.

[7] Barro, R., & Sala-i-Martin, X. (2004). Economic growth, second edition.

[8] Bureau of Economic Analysis, 2003. Fixed Assets and Consumer Durable Goods in the United States, 1925–99. Washington, DC: U.S. Government Printing Office.

[9] Bertola, G. (1993). Factor Shares and Savings in Endogenous Growth. The American Economic Review, 1184-1198.

[10] Bertola, G. (1996). Factor shares in OLG models of growth. European Economic Review, 40(8), 1541-1560.

[11] Best, M. C., Cloyne, J. S., Ilzetzki, E., & Kleven, H. J. (2020). Estimating the elasticity of intertemporal substitution using mortgage notches. The Review of Economic Studies, 87(2), 656-690.

[12] Borio, Claudio E.V. and Disyatat, Piti and Juselius, Mikael and Rungcharoenkitkul, Phurichai, 2017. Why So Low for So Long? A Long-Term View of Real Interest Rates. Bank of International Settlements Working Paper No. 685.

[13] Caballero, R. J., Farhi, E., & Gourinchas, P. O. (2017). Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares. American Economic Review, 107(5), 614-20.

[14] Cass, D., 1965. Optimum Growth in an Aggregative Model. Review of Economic Studies 32, 233-40.

[15] Cooley, T. F., & Prescott, E. C. (1995). Economic Growth and Business Cycles. In Frontiers of business cycle research (pp. 1-38). Princeton University Press.

[16] Dalgaard, C. J., & Jensen, M. K. (2009). Life-cycle savings, bequest, and a diminishing impact of scale on growth. Journal of Economic Dynamics and Control, 33(9), 1639-1647.

[17] DeLong, J. B. (2003). Bequests: an historical perspective. The Role and Impact of Gifts and Estates, Brookings Institution.

[18] Diamond, P., 1965. National Debt in a Neoclassical Growth Model. American Economic Review 55, 5, 1126-55

[19] De Nardi, M., French, E., & Jones, J. B. (2016). Savings after retirement: A survey. Annual Review of Economics, 8, 177-204.

[20] Dynan, K. E., Skinner, J., & Zeldes, S. P. (2002). The importance of bequests and life-cycle saving in capital accumulation: A new answer. American Economic Review, 92(2), 274-278.

[21] Eggertsson, G. B., Robbins, J. A., & Wold, E. G. (2021). Kaldor and Piketty's facts: The rise of monopoly power in the United States. Journal of Monetary Economics, 124, S19-S38.

[22] Farhi, E., & Gourio, F. (2018). Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia. Brookings Papers on Economic Activity, 147.

[23] Feenstra, Robert C., Robert Inklaar and Marcel P. Timmer (2015), "The Next Generation of the Penn World Table" American Economic Review, 105(10), 3150-3182, available for download at www.ggdc.net/pwt

[24] Fraumeni, B. (1997), "The Measurement of Depreciation in the U.S. National Income and Product Accounts", Survey of Current Business, Vol 77.

[25] Galor, O. (1996). Convergence? Inferences from theoretical models. The Economic Journal, 106(437), 1056-1069.

[26] Galor, O., & Ryder, H. E. (1989). Existence, uniqueness, and stability of equilibrium in an overlapping-generations model with productive capital. Journal of Economic Theory, 49(2), 360-375.

[27] Galor, O. (1992). A two-sector overlapping-generations model: A global characterization of the dynamical system. Econometrica: Journal of the Econometric Society, 1351-1386.

[28] Gomme, P., Ravikumar, B., & Rupert, P. (2011). The return to capital and the business cycle. Review of Economic Dynamics, 14(2), 262-278.

[29] Hall, R. E. (1968). Technical Change and Capital from the Point of View of the Dual. Review of Economic Studies, 35(1), 35-46.

[30] Hémous, D. & Olsen, M. (2022). The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality. American Economic Journal: Macroeconomics, 14(1), 179-223.

[31] Hulten, C.R. and Wykoff, FC. (1981) The estimation of economic depreciation using vintage asset prices: An application of the Box-Cox power transformation, Journal of Econometrics, 15(3).

[32] Inklaar, R., Woltjer, P., & Albarrán, D. G. (2019). The composition of capital and cross-country productivity comparisons. International Productivity Monitor, (36), 34-52.

[33] Jones, L. E., & Manuelli, R. E. (1992). Finite lifetimes and growth. Journal of Economic Theory, 58(2), 171-197.

[34] Jones, C. I. (2016). The facts of economic growth. In Handbook of macroeconomics (Vol. 2, pp. 3-69). Elsevier.

[35] Karabarbounis, L. & Neiman, B. (2014). The global decline of the labor share. Quarterly Journal of Economics, 129(1), 61-103.

[36] Katz, A., & Herman, S. W. (1997). Improved estimates of fixed reproducible tangible wealth, 1929-95.

[37] Kiley, M. T. (2020). The global equilibrium real interest rate: concepts, estimates, and challenges. Annual Review of Financial Economics, 12, 305-326.

[38] Koopmans, T.C., 1965. On the concept of Optimal Economic Growth. The Economic Approach to Development Planning. Chicago: Rand McNally, 225-87.

[39] Kotlikoff, L. J., & Summers, L. H. (1981). The role of intergenerational transfers in aggregate capital accumulation. Journal of political economy, 89(4), 706-732.

[40] Lunsford, K. G., & West, K. D. (2019). Some evidence on secular drivers of US safe real rates. American Economic Journal: Macroeconomics, 11(4), 113-39.

[41] Modigliani, F. (1988). The role of intergenerational transfers and life cycle saving in the accumulation of wealth. Journal of Economic Perspectives, 2(2), 15-40.

[42] Moll, B. & Rachel, L. & Restrepo, P. (2021). Uneven Growth: Automation's Impact on Income and Wealth Inequality, Working Paper

[43] Oulton, N., & Srinivasan, S. (2003). Capital stocks, capital services, and depreciation: An integrated framework. Bank of England. Quarterly Bulletin, 43(2), 227.

[44] Phelps, E. (1962). The New View of Investment: A Neoclassical Analysis. The Quarterly Journal of Economics, 76(4), 548-567.

[45] Piketty, T. (2011). On the long-run evolution of inheritance: France 1820--2050. The quarterly journal of economics, 126(3), 1071-1131.

[46] Uhlig, H., & Yanagawa, N. (1996). Increasing the capital income tax may lead to faster growth. European Economic Review, 40(8), 1521-1540.

[47] Sajedi, R., & Thwaites, G. (2016). Why are real interest rates so low? the role of the relative price of investment goods. IMF Economic Review, 64(4), 635-659.

[48] Schmelzing, P. Eight centuries of global real rates and the suprasecular decline, 1311-2018. November 2019. Manuscript, Harvard University.

[49] Rachel, L., & Smith, T. D. (2017). Are low real interest rates here to stay?. International Journal of Central Banking, 13(3), 1-42.

[50] Rachel, L. & Summers (2019). On falling neutral real rates, fiscal policy and the risk of secular stagnation. In Brookings Papers on Economic Activity BPEA Conference Drafts, March 7-8.

[51] Ramey, V. A., & Shapiro, M. D. (2001). Displaced capital: A study of aerospace plant closings. Journal of political Economy, 109(5), 958-992.

[52] Ramsey, F., 1929. A Mathematical Theory of Savings. Economic Journal 38, 543-59.

[53] Whelan, K. (2002). A guide to US chain aggregated NIPA data. Review of income and wealth, 48(2), 217-233.

[54] Yaari, M. E. (1964). On the consumer's lifetime allocation process. International Economic Review, 5(3), 304-317.

Appendix

The real rate and depreciation: standard models

To begin consider a **Solow-Swan** model where production is Cobb-Douglas, $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$. All markets are competitive and time is discrete. Since savings are generated as a constant fraction (s) of total income, and the economy is closed, $K_{t+1} = sY_t + (1-\delta)K_t$. Population grows at the constant rate n and technological progress occurs at the rate g. In the steady state the model predicts that the capital-output ratio converges to

$$\left(\frac{k}{y}\right)^* = \frac{s}{n+g+\delta}$$

where k and y are both expressed in efficiency units $(x = X/AL)$. Assuming competitive markets, the real rate of return is given by the first-order condition from firm profit maximization

$$r_t = \alpha\frac{y_t}{k_t} - \delta. \tag{10}$$

In the steady state the real rate of interest therefore approaches

$$\lim_{t\to\infty} r_t = r^* = \alpha\frac{n+g+\delta}{s} - \delta.$$

As can be seen, the impact on r^* from a change in the depreciation rate depends on the ratio of α to s. If $\alpha = s$, which means the economy is at the Golden Rule steady state, the steady-state real rate of return is exactly independent of the depreciation rate. Only if $\alpha < s$, i.e. if the economy is dynamically inefficient, does an increase in capital depreciation lower r^*. Absent dynamic inefficiency, a Solow-Swan model would therefore predict that a higher depreciation rate will work to *increase* the real rate of interest. The reason is simple. Depreciation has two separate effects on r. First, an increase in δ directly lowers the net marginal product of capital. Second, it indirectly reduces the long-run capital-output ratio and thus increases the long-run gross marginal product. In the golden rule steady state, the two countervailing forces exactly offset, whereas, for a dynamically efficient economy, the reduction in the long-run capital-output ratio dominates.

Consider next a **Ramsey-Cass-Koopmans** model (RCK; Ramsey, 1929; Cass, 1965; Koopmans, 1965). As noted in Section 3 this environment arises in an overlapping generations setting when households are sufficiently altruistic (Barro, 1974). If the household values future generations sufficiently it chooses to pass on bequest and the economy becomes isomorphic to one populated by "infinitely living agents". Alternatively bequests are not passed on and the economy "collapses" to a standard Diamond model.

Without loss, assume per period utility is logarithmic for simplicity. In the RCK model the real rate is pinned down by the first order condition assigned to the problem of maximizing utility from consumption over an infinite horizon, the consumption Euler:

$$\frac{c_{t+1}}{c_t} = r_t - \rho \Rightarrow r^* = g + \rho. \tag{11}$$

Clearly, in a RCK model, δ does not affect r^*. The reason is that along a steady-state path, the Ramsey consumer reacts to a higher depreciation rate by adjusting savings downward. As a result, a change in δ leaves the net marginal product of capital constant and with it r^*. The RCK framework thus holds the same steady-state prediction as a Solow-Swan model, evaluated at the golden-rule steady state.[27]

Finally, consider a **Diamond** (1965) model. Production is Cobb-Douglas, and preferences are (for now) logarithmic, which means the savings rate of the young is independent of the real rate of return. Optimal savings of the young is given by $s_t w_t$ where $s_t = 1/(2+\rho)$ and w is (lifetime) wage income. The capital stock in period $t+1$ is given by the savings of the young in period t, $K_{t+1} = s_t w_t L_t$. As a result, the law of motion for capital per efficiency unit of labor, $k \equiv K/(AL)$, takes the form

$$k_{t+1} = \frac{(1-\alpha)}{(1+g)(1+n)(2+\rho)} k_t^{\alpha},$$

where we have used that $w_t = (1-\alpha)k_t^{\alpha}$ from the firm first order conditions. In the steady state (which is unique and stable) the capital-output ratio approaches:

$$\left(\frac{k}{y}\right)^* = \frac{1-\alpha}{(1+g)(1+n)(2+\rho)}.$$

The capital stock, and with it output, remains unchanged by higher δ because the capital stock in period $t+1$ is purely a function of savings by the young in period t which depends on wage income i period t. Since the capital stock is productive before any depreciation happens, a higher depreciation rate only translates into a lower capital income which flows to the (non-saving) old in period t.

Substituting for $(k/y)^*$ in the first order condition for capital (equation 10) we obtain the prediction for the long run real rate of interest:

$$r^* = \alpha \frac{(1+g)(1+n)(2+\rho)}{1-\alpha} - \delta. \tag{12}$$

In this version of the Diamond model, a higher δ does not affect the gross marginal product in the steady state, and therefore translates 1:1 into a *lower* r^*. Observe that this result holds whether the economy is dynamically inefficient or not.

Finally, consider a **Diamond model with more general preferences**

$$u = \frac{c_{1t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \frac{c_{2t+1}^{1-\theta} - 1}{1-\theta}$$

The savings rate is now a function of the (future) real rate of interest, $s\left(r_{t+1}\right)$. For existence and uniqueness of a self-fulfilling expectation path we require (cf Galor and Ryder, 1989, lemma 1):

$$s_r \geq 0.$$

[27]the RCK model does not suggest δ is completely irrelevant, as changes in δ will affect r in transition. Suppose, for example, that δ rises from δ_0 to $\delta_1 > \delta_0$ in a RCK model. This will lower the long-run capital stock in efficiency units. However, in the short run, where the capital stock is fixed, the real rate of return is predicted to fall, after which it increases monotonically in transition to the new steady state where its level is the same as before the shock. A standard parameterization of the RCK model would suggest that the economy closes about 4 - 5% of the gap to the steady state per year (e.g., Barro and Sala-i-Martin, 2004), implying that half the gap to the steady state is closed within 14 to 18 years. But since the real rate appears to decline systematically over the long run transitional dynamics are unlikely to be able to fully account for the data.

The law of motion of the capital stock in efficiency units (since we maintain Cobb-Douglas production) is

$$k_{t+1} = \frac{1}{(1+n)(1+g)} s(r_{t+1})(1-\alpha) k_t^{\alpha},$$

which means that we can write the steady state

$$\left(\frac{k}{y}\right)^* = \frac{(1-\alpha)}{(1+n)(1+g)} s(r^*),$$

since the law of motion is fulfilled at all points in time, the steady state included. This gives combinations of the capital-output ratio and the real rate of interest that are consistent with a (unique) steady state. The schedule is monotonically upward sloping due to the requirement imposed by existence and uniqueness self-fulfilling expectation path,$s_r \geq 0$ for all r. The intuition is that a higher real rate of interest spurs savings and fosters a higher capital-output ratio in the steady state.

To provide comparative statics wrt. to the real rate of interest we need the FOC from the firm

$$r = \alpha \left(\frac{k}{y}\right)^{-1} - \delta$$

which gives combinations of capital-output ratios and the real rate that are consistent with clearing in the market for capital. This equation is downward sloping implying a unique intersection point between the two schedules.

Plotting the two schedules in a $(K/Y, r)$ diagram, and perturbing the depreciation rate, immediately leads to the insight that in the steady state the capital-output ratio is lowered, but so is the real rate of interest. Hence, the downward adjustment in the capital stock does not fully compensate for the direct impact of a rising depreciation rate.